Correspondence

A General Framework for Frequency Domain Multi-Channel Signal Processing

A. K. Katsaggelos, K. T. Lay, and N. P. Galatsanos

Abstract—**In this paper, we provide a general framework for performing linear shift-invariant within channel and shift varying across channels processing of stationary multi-channel (MC) signals. Emphasis is given on the restoration of degraded signals. We show that, by utilizing the special structure of semi-block circulant and block diagonal matrices, MC signal processing can be easily carried out in the frequency domain. The generalization of many frequency domain single-channel (SC) signal processing techniques to the MC case is presented. We show that in MC signal processing each frequency component of a signal and system is respectively represented by a small vector and a matrix (of size equal to the number of channels), while in SC signal processing each frequency component in both cases is a scalar.**

I. Introduction

A multi-channel (MC) signal refers to a set of signals that exhibit cross-channel similarity or correlation. When an MC signal is processed by a system, signals in different channels affect each other in producing the output due to cross-channel correlations. Among the MC signals of special interest are MC images, such as color images, sequences of images, and the channels resulting from the wavelet-based decomposition of a single-channel image [2], [3].

One of the important problems in signal processing is the restoration of degraded signals. Various filters have been successfully designed for the restoration of single-channel (SC) signals [1], [11]. The direct application of SC techniques to MC processing, however, has not been very successful, due primarily to the large and complicated matrices involved in computations. Nevertheless, the restoration of MC images has been a quite active research topic in the past decade. *Hunt* and *Kubler* [9] applied the Karhunen–Loeve transformation to decompose the MC Wiener filter into SC Wiener filters. *Galatsanos* and *Chin* [5] removed the constraints in *Hunt* and *Kubler's* work and proposed a recursive algorithm for the efficient inversion of large matrices required for MC restoration. In *Galatsanos et al.* [6] regularized least-squared MC restoration filters were developed and a nonrecursive matrix inversion algorithm was also proposed. Kalman MC image restoration filters were developed by *Tekalp* and *Pavlovic* [15] and *Galatsanos* and *Chin* [7]. *Katsaggelos* [10] derived adaptive restoration algorithms based on a set theoretical regularization technique, for the case when all channels are identical. Such a case results, for example, when the frames of a sequence are compensated for the motion.

In this paper, we provide a general framework for performing linear filtering of MC signals [13]. The underlying assumption is that the linear system relating the i-th input channel to the j-th output channel is shift-invariant, characterized by the impulse response h_{ij}. However, the linear system is not shift-invariant across channels, i.e., $h_{ij} \neq h_{i+k,j+k}$. A similar assumption is used for the covariance matrix of the MC signal. In other words, the auto- and cross-covariance for all channels are stationary, although $R_{ij} \neq R_{i+k,j+k}$, where R_{ij} is the covariance matrix of channels i and j. We also extend many frequency domain SC signal processing techniques to the MC case. We show that discrete frequency domain processing is identical in both cases, if each scalar frequency component of a SC signal and system is respectively replaced by a vector and a matrix for the MC case.

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A. K. Katsaggelos is with Northwestern University, Dept. of Electrical Engineering and Computer Science, Evanston, IL 60208.

K. T. Lay is with the National Taiwan Inst. of Tech., Dept. of Electrical Engineering and Computer Science, Taipei, Taiwan, R.O.C.

N. P. Galatsanos is with the Illinois Institute of Technology, Dept. of Electrical and Computer Engineering, Chicago, IL 60616.

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II. Special Matrix Structures in Multi-Channel Signal Processing

Let us consider a P-channel MC signal x, where each channel has N samples.[1] We choose to represent such a signal x in a vector form as follows:

$$x = [x_1(0)x_2(0)\cdots x_P(0)x_1(1)x_2(1)\cdots x_P(1) \cdots x_1(N-1)x_2(N-1)\cdots x_P(N-1)]^T \tag{1}$$

where T denotes the transpose of a matrix or a vector. This representation in novel and leads to the natural extension of SC frequency domain signal processing techniques to MC signals, as shown in the rest of the paper. It results in semi-block circulant (SBC) matrices when vector-matrix representation of the linear systems is used.

A SBC matrix is a square matrix of the form

$$A = \begin{bmatrix} A_{11} & A_{12} & \dots & A_{1N} \\ A_{1N} & A_{11} & \dots & A_{1,N-1} \\ \dots & \dots & \dots & \dots \\ A_{12} & A_{13} & \dots & A_{11} \end{bmatrix} \tag{2}$$

that is, it is circulant at the block level but each A_{1j} is an arbitrary $P \times P$ matrix, $1 \leq j \leq N$. For convenience, let us refer to the order of A as (P, N). For brevity, the notation $A \in SBC(P, N)$ is used to denote that A is a SBC matrix of order (P, N).

An example where a SBC matrix is encountered is given by the MC degradation model [6], [7]

$$y_i(m) = \sum_{j=1}^{N} \sum_{p \in S_{D_{ij}}} d_{ij}(p) x_j(m-p) + v_j(m), \quad 1 \leq i \leq P \tag{3}$$

where $S_{D_{ij}}$ is the support region of $d_{ij}(m)$, and $d_{ij}(m) \neq d_{i+k,j+k}(m)$. Expressed in a matrix/vector form, (3) is rewritten as

$$y = Dx + v \tag{4}$$

where the vectors y, x, and n are formed according to (1). Since the discrete Fourier transform (DFT) will be used in implementing convolution, we assume that (3) represents circular convolution. A sequence can be padded with zeros in such a way that the result of

[1] The presentation is in terms of one-dimensional MC signals for notational convenience. However, the results apply directly to MC multi-dimensional signals.

linear and circular convolution is the same [3] or the observed signal can be preprocessed around its boundaries so that (3) is consistent with the circular convolution of $\{d_{ij}(p)\}$ with $\{x_j(p)\}$. In this case $D \in SBC(P, N)$, as is easily verified.

In the rest of this section we describe and analyze the basic properties of an SBC matrix, since this will allow us to move to the discrete frequency domain. The fundamental property of an SBC matrix that allows the extension of frequency domain SC signal processing techniques to MC signals is given by the following theorem.

Theorem 1: An SBC matrix A can be reduced to a block diagonal (BD) matrix B by the similarity transformation, $A = \Omega B \Omega^{-1}$, where B is given by

$$B = \begin{bmatrix} B_{11} & 0 & \cdots & 0 \\ 0 & B_{22} & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & B_{NN} \end{bmatrix} \tag{5}$$

with B_{ii} a $P \times P$ matrix, $1 \le i \le N$, and

$$\Omega = \frac{1}{N} \begin{bmatrix} \epsilon_{00} I_P & \epsilon_{01} I_P & \cdots & \epsilon_{0,N-1} I_P \\ \epsilon_{10} I_P & \epsilon_{11} I_P & \cdots & \epsilon_{1,N-1} I_P \\ \cdots & \cdots & \cdots & \cdots \\ \epsilon_{N-1,0} I_P & \epsilon_{N-1,1} I_P & \cdots & \epsilon_{N-1,N-1} I_P \end{bmatrix} \tag{6}$$

with I_P a $P \times P$ identity matrix and $\epsilon_{pq} = \exp\{j(2\pi pq)/N\}$.

The above theorem is shown in a straightforward way by decomposing $\Omega^{-1} A \Omega = B$ into $E^{-1} A(i, j) E = B(i, j)$, $i, j = 1, 2, \cdots, P$, where $A(i, j)$ and $B(i, j)$ are $N \times N$ matrices containing the (i, j) elements of A_{1k} and $B_{\ell k}$, $\ell, k = 1, 2, \cdots, N$, and

$$E = \frac{1}{N} \begin{bmatrix} \epsilon_{00} & \epsilon_{11} & \cdots & \epsilon_{0,N-1} \\ \epsilon_{10} & \epsilon_{11} & \cdots & \epsilon_{1,N-1} \\ \cdots & \cdots & \cdots & \cdots \\ \epsilon_{N-1,0,} & \epsilon_{N-1,1} & \cdots & \epsilon_{N-1,N-1} \end{bmatrix}.$$

It is easy to verify that $E^{-1} = E^H$, where H denotes the Hermitian of a matrix or a vector, since E^H is the discrete Fourier transform matrix. Furthermore, $A(i, j)$ is circulant since A is circulant at the block level. Therefore, $B(i, j)$ is diagonal and thus B is block diagonal.

For convenience, the order of B is referred to as (P, N) and the sub-block B_{kk} is called the k-th component of B. For brevity, $B(k)$ is interchangeably used with B_{kk} and the notation $B \in BD(P, N)$ is used to denote that B is a BD matrix of order (P, N).

Our interest now shifts to the algebraic operations on BD matrices. Some useful properties about these operations are presented next.

Property 1: Given $B \in BD(P, N)$, with $B(k)$ the k-th component matrix, $0 \le k \le N-1$, B^{-1} exists if and only if $B(k)^{-1}$ exists for $0 \le k \le N-1$. In the case that B^{-1} exists, $B^{-1} \in BD(P, N)$ and can be computed component by component by

$$B^{-1}(k) = B(k)^{-1} \tag{7}$$

where $B^{-1}(k)$ and $B(k)$ are respectively the k-th components of B^{-1} and B. This property is verified by showing that the product of B and the BD matrix with the k-th component given by (7) is an identity matrix. Since the inverse of a matrix is unique, this verification completes the proof. This property is also shown in [6] (Appendix C) and then used in [14].

Property 2: Given $B \in BD(P, N)$, with $B(k)$ the k-th component matrix, $0 \le k \le N-1$, the determinant of B can be computed by

$$\det(B) = \det(B(1)) \cdot \det(B(2)) \cdot \cdots \cdot \det(B(N)). \tag{8}$$

Property 3: Given $B \in BD(P, N)$, with $B(k)$ the k-th component matrix $0 \le k \le N-1$, suppose that B has eigenvalues $\{\lambda_1, \lambda_2, \cdots, \lambda_{NP}\} \triangleq \mathrm{EigVal}(B)$ and $B(k)$ has eigenvalues $\{\lambda(k)_1, \lambda(k)_2, \cdots, \lambda(k)_P\} \triangleq \mathrm{EigVal}(B(k))$, $0 \le k \le N-1$. Then

$$\mathrm{EigVal}(B) = \bigcup_{k=0}^{N-1} \mathrm{EigVal}(B(k)) \tag{9}$$

where $\cup$ denotes set union.

Proof: $\mathrm{EigVal}(B)$ is obtained by solving $\det(\lambda B - I_{NP}) = 0$ [8] (I_{NP} is an $NP \times NP$ identity matrix), which is equivalent to solving $\det(\lambda B(k) - I_P) = 0$, $0 \le k \le N-1$ (by Prop. 2), which in turn determines $\mathrm{EigVal}(B(k))$.

Property 4: Given $B \in BD(P, N)$, with $B(k)$ the k-th component matrix, $0 \le k \le N-1$, suppose that B has singular values $\{\sigma_1, \sigma_2, \cdots, \sigma_{NP}\} \triangleq \mathrm{SingVal}(B)$ and $B(k)$ has singular values $\{\sigma(k)_1, \sigma(k)_2, \cdots, \sigma(k)_P\} \triangleq \mathrm{SingVal}(B(k))$, $0 \le k \le N-1$. Then

$$\mathrm{SingVal}(B) = \bigcup_{k=0}^{N-1} \mathrm{SingVal}(B(k)). \tag{10}$$

Proof: Recall that the singular values of B are the square roots of the intersection of the nonzero eigenvalues of BB^H and $B^H B$ [8]. Then, it is clear that this property follows from Prop. 2 and Prop. 3.

Property 5: Addition and multiplication of $BD(P, N)$ matrices results in a $BD(P, N)$ matrix.

III. Frequency-Domain Framework

With the mathematical background provided in the previous section, we now present a general framework for MC signal filtering/restoration in the discrete frequency domain. First we define the frequency component of an MC signal and show that MC filtering can be performed component by component. Then the MC Wiener filter and the singular value decomposition (SVD) of a SBC matrix in the context of pseudo-inverse filtering are discussed. Towards this end the following two definitions are required.

Definition: Given a vector sequence $\{x(n), 0 \le n \le N-1\}$, where $x(n)$ is a P-vector (or equivalently, a $P \times 1$ array), the vector discrete Fourier transform (V-DFT) of $\{x(n)\}$ is equal to

$$X(k) = \sum_{n=0}^{N-1} x(n) \exp\left(\frac{-j2\pi nk}{N}\right), \quad 0 \le k \le N-1. \tag{11}$$

Definition: Given an array sequence $\{A(n), 0 \le n \le N-1\}$, where $A(n)$ is a $P \times P$ array, the array discrete Fourier transform (A-DFT) of $\{A(n)\}$ is equal to

$$A_{DFT}(k) = \sum_{n=0}^{N-1} A(n) \exp\left(\frac{-j2\pi nk}{N}\right), \quad 0 \le k \le N-1. \tag{12}$$

A. Multi-Channel Filtering in the Frequency Domain

As explained in the introduction, MC filtering is expressed in the spatial domain in matrix/vector form by $y = Gx$, where G is a SBC matrix. G is reducible to the BD matrix Θ_G according to

$$G = \Omega \Theta_G \Omega^{-1} \tag{13}$$

where Ω is given by (6). Note that $\Theta_G(k)$, $0 \le k \le N-1$ is obtained by taking the array discrete Fourier transform (A-DFT) of the first row submatrices of G. Since the first row submatrices of G (i.e., $\{G_{1m}, 0 \le m \le N-1\}$) consist of the system's impulse response $\{g_{ij}(m), 1 \le i, j \le P, 0 \le m \le N-1\}$, $\{\Theta_G(k)\}$ consists of the within-channel and cross-channel frequency responses. For the input signal x its k-th frequency component is defined as

$$X(k) = [X_1(k)X_2(k)\cdots X_P(k)]^T, \quad 0 \le k \le N-1 \tag{14}$$

where X is the V-DFT of x. $Y(k)$, the k-th component of the output signal $Y(k)$ is defined in a similar way. With the MC frequency component defined above, MC filtering is written component by component as

$$Y(k) = \Theta_G(k)X(k), \quad 0 \le k \le N-1. \tag{15}$$

Equation (15) represents the extension of the frequency domain SC filtering to the MC case. Note that in the MC case the frequency component of a signal is now a $P \times 1$ matrix and the frequency component of the transfer function is a $P \times P$ matrix, instead of a scalar.

B. Multi-Channel Wiener Filter

In further demonstrating the applicability of the material in Section II. We consider here the frequency domain expression for the MC Wiener filter. Based on the degradation model of (4), a restored signal $\hat{x}$ by the MC Wiener filter is given by [1]

$$\hat{x} = R_{XX}D^H\Big[DR_{XX}D^H + R_{VV}\Big]^{-1}y. \tag{16}$$

R_{XX} and R_{VV} are the covariance matrices of x and v, respectively, which are SBC matrices because x_i and v_i in (3) are assumed to be stationary within each channel. The computation of (16) requires the inversion of the matrix $\Phi = [DR_{XX}D^H + R_{VV}]$, which is difficult due to the large order of Φ ($PN \times PN$, to be exact).

We are now in the position of obtaining a discrete frequency domain expression for the Wiener filter in (16). Since D, R_{XX} and R_{VV} are SBC matrices, Φ can be written as

$$\Phi = \Omega\Big(\Theta_D\Theta_{XX}\Theta_D^H + \Theta_{VV}\Big)\Omega^{-1}. \tag{17}$$

where Θ_D, Θ_{XX} and Θ_{VV} are the corresponding BD matrices according to theorem 1. For easy referencing, let us define

$$\Psi = \Big(\Theta_D\Theta_{XX}\Theta_D^H + \Theta_{VV}\Big). \tag{18}$$

Θ_D consists of the frequency response of the within-channel and the cross-channel degradation. Θ_{XX} is obtained from block-diagonizing R_{XX} by taking the A-DFT of its first row submatrices. Since R_{XX} consists of the within-channel and the cross-channel correlation, the A-DFT of its first row submatrices represents the power and cross spectra of the original signal. Similarly, the elements of Θ_{VV} represent the power and cross spectra of the additive MC noise v.

Due to Property 5, Ψ is also a BD matrix. Its k-th component is equal to

$$\Psi(k) = \Theta_D(k)\Theta_{XX}(k)\Theta_D(k)^H + \Theta_{VV}(k). \tag{19}$$

Equation (16) can now be written as

$$\hat{x} = \Omega\Theta_{XX}\Theta_D^H\Psi^{-1}\Omega^{-1}y = \Omega\Theta_{XX}\Theta_D^H\Psi^{-1}Y \tag{20}$$

where

$$Y = \Omega^{-1}y = [Y(0)Y(1)\cdots Y(N-1)]^T \tag{21}$$

with $Y(k) = [Y_1(k)Y_2(k)\ldots Y_P(k)]^T$ the V-DFT of the observed signal y. Premultiplying (21) by Ω^{-1}, we have

$$\hat{X} = \Theta_{XX}\Theta_D^H\Psi^{-1}Y \tag{22}$$

where $\hat{X} = \Omega^{-1}\hat{x}$ is the V-DFT of the restored signal $\hat{x}$.

Noting that all operations in (22) can be carried out component by component, it is written as

$$\hat{X}(k) = \Theta_{XX}(k)\Theta_D(k)^H\Psi^{-1}(k)Y(k), \quad 0 \le k \le N-1 \tag{23}$$

where $\hat{X}(k)$ is a $P \times 1$ vector shown in (14). From (23), we see that the computational complexity for obtaining $\hat{X}(k)$, $0 \le k \le N-1$, is $N \cdot O(P^3)$, which is usually manageable since in most applications $P \ll N$.

C. Multi-Channel Singular Value Decomposition

As another application of the material presented in Section II consider the singular value decomposition (SVD) of an SBC matrix. This is, for example, a required step in MC pseudo-inverse filtering. That is, consider again (4), when v is ignored. The minimum norm least-squares solution $\hat{x}^+$ given by

$$\hat{x}^+ = D^+y \tag{24}$$

where D^+, the pseudo-inverse of D, is also an SBC (P, N) matrix. Then (24) is rewritten as

$$\hat{x}^+ = \Omega\Theta_{D^+}\Omega^{-1}Y \quad \text{or} \quad \hat{X}^+ = \Theta_{D^+}Y \tag{25}$$

where $\hat{X}^+$ and Y are the V-DFT of $\hat{x}^+$ and y, respectively, and Θ_{D^+} is a BD (P, N) matrix with elements $\Theta_{D^+}(k)$.

In computing Θ_{D^+} the SVD of $\Theta_D = \Omega^{-1}D\Omega$ is required. It is given by

$$\Theta_D = Q_1\Sigma_{\Theta_D}Q_2^H \tag{26}$$

where Σ_{Θ_D} is a $PN \times PN$ diagonal matrix with entries the singular values of Θ_D, and Q_1 and Q_2 are BD orthogonal matrices formed by the singular vectors. According to Property 4 in Section II, $\mathrm{SingVal}(\Theta_D) = \cup_{k=0}^{N-1}\mathrm{SingVal}(\Theta_D(k))$. That is, the singular values of Θ_D are obtained by the SVD of the $P \times P$ component matrices $\Theta_D(k)$, $0 \le k \le N-1$, according to

$$\Theta_D(k) = Q_1(k)\Sigma_{\Theta_D(k)}Q_2^H(k), \quad 0 \le k \le N-1 \tag{27}$$

where $\Sigma_{\Theta_D(k)}$ is a $P \times P$ diagonal matrix with elements the singular values of $\Theta_D(k)$. The $P \times P$ matrices $Q_1(k)$ and $Q_2(k)$ are orthogonal and are formed by the singular vectors. They are the diagonal entries of the BD (P, N) matrices Q_1 and Q_2 in (26). That is, the decomposition represented by (26) is broken down in N decompositions, according to (27). Θ_{D^+} is then defined by

$$\Theta_{D^+} = Q_2\Sigma_{\Theta_D^+} + Q_1^H \tag{28}$$

where all matrices are BD (P, N). The $P \times P$ elements $\Theta_{D^+}(k)$ of Θ_{D^+} are given by

$$\Theta_{D^+}(k) = Q_1(k)\Sigma_{\Theta_{D^+}(k)}Q_2^H(k), \quad 0 \le k \le N-1 \tag{29}$$

where the entries of $\Sigma_{\Theta_{D^+}(k)}$, and equal to zero if the entry of $\Sigma_{\Theta_D(k)}$ is zero.

Therefore, (25) can be rewritten component by component as

$$\hat{X}^+(k) = \Theta_{D^+}(k)Y(k), \quad 0 \le k \le N-1. \tag{30}$$

If $\Theta_D(k)$ is nonsingular, then $\Theta_{D^+}(k) = \Theta_{D^{-1}}(k) = [\Theta_D(k)]^{-1}$. Similarly, D^+ is equal to $D^+ = \Omega Q_2\Sigma_{\Theta_{D^+}}Q_1^H\Omega^{-1}$. Again, the main result of this section is that since only small $(P \times P)$ matrices are involved in the SVD of a SBC matrix, the computation is simple.

IV. Conclusions

In this paper we have presented the mathematical framework that relates SC to P-channel MC signal processing in the frequency domain. This framework is based on the key result of the equivalence between SBC and BD matrices. It was shown that using this framework, each frequency component of the frequency response of an MC linear system is represented by a $P \times P$ matrix. Thus, in contrast to the SC case where an N-component signal processing in the frequency domain requires N scalar operations, for the MC P-channel case it requires N matrix/vector operations involving $(P \times P) \times (P \times 1)$ elements. This increases the computational load by $O(P^3)$, but this increase is not prohibitive because $P \ll N$ in most cases.

Using this framework, it is straightforward to verify that MC image processing algorithms that have appeared in the literature [2], [5]–[7], [12], [15], [16] can be derived directly from their SC counterparts.

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