Decision Directed Algorithms for Multiuser Detection

George V. Moustakides[1]
Dept. of Computer Eng. & Informatics
University of Patras, Greece.
E-mail: moustaki@ceid.upatras.gr

Abstract — **We present a class of constraint LMS like adaptive linear detection schemes that constitutes a generalization to the popular blind adaptive detector. We show that, contrary to the general belief, the conventional LMS and its constraint version, when in training mode, do not necessarily outperform the blind LMS of [1]. Trained algorithms uniformly outperform their blind counterparts only if they incorporate knowledge of the amplitude of the user of interest. Decision directed versions of such algorithms are shown to be equally efficient as their trained prototypes and significantly better than the blind versions.**

Background and Main Results

A K-user synchronous DS-CDMA system with identical chip waveforms and signaling antipodally through additive white Gaussian noise (AWGN) channel, in discrete time can be modeled as

$$\mathbf{r}(n) = \sum_{i=1}^{K} \mathbf{s}_i a_i b_i(n) + \sigma \mathbf{n}(n).$$

where $\mathbf{r}(n)$ is the received vector of length N, N the spreading factor; σ^2 the power of the AWGN, $\mathbf{s}_i$ the signature of User-i, a_i the corresponding amplitude, $b_i(n)$ the n-th symbol of User-i and finally $\mathbf{n}(n)$ a white Gaussian noise vector with i.i.d. components of zero mean and unit variance.

Linear detectors estimate the transmitted bits by taking $\hat{b}_i(n) = \text{sgn}\{\mathbf{c}^t\mathbf{r}(n)\}$ with $\mathbf{c}$ a suitable filter. Since the last estimate is also equal to $\text{sgn}\{\delta\mathbf{c}^t\mathbf{r}(n)\}$ when $\delta > 0$, we conclude that there is an ambiguity in $\mathbf{c}$ as far as detection is concerned. This ambiguity can be eliminated by imposing a constraint on the filter, and we propose the use of the same constraint as the one in [1], namely $\mathbf{c}^t\mathbf{s}_1 = 1$.

The constraint stated above can now be used to define the following constraint mean square error minimization problem

$$\min_{\mathbf{c}} E\{(v(n) - \mathbf{c}^t\mathbf{r}(n))^2\}, \quad \mathbf{c}^t\mathbf{s}_1 = 1,$$

where $v(n)$ is a process to be specified next. An LMS like stochastic gradient algorithm that solves the above problem adaptively can be defined as follows

$$\begin{aligned} \epsilon(n) &= v(n) - \mathbf{c}^t(n-1)\mathbf{r}(n), \quad \mathbf{c}(0) = \mathbf{s}_1, \\ \mathbf{c}(n) &= \mathbf{c}(n-1) + \mu\epsilon(n)(\mathbf{r}(n) - \mathbf{s}_1^t\mathbf{r}(n)\mathbf{s}_1). \end{aligned}$$

As far as $v(n)$ is concerned we are interested in two selections, namely $v(n) = \alpha b_1(n)$ and $v(n) = \alpha\text{sgn}\{\mathbf{c}^t(n-1)\mathbf{r}(n)\}$, where α a scalar parameter. The first results in a trained LMS like algorithm and the second in its corresponding decision directed (DD) version. Notice that $\alpha = 0$ yields the blind LMS (BLMS) of [1]; $\alpha = 1$ the constraint LMS (CLMS) and $\alpha = a_1$, a constraint LMS with amplitude information (CLMS-AI).

To examine the behavior of the algorithms, we use the excess inverse signal to interference ratio as our performance measure and let us denote it as $J(n)$. We have the following results concerning the trained version for any α and the DD version for $\alpha = a_1$ (subscripts t and d respectively).

Theorem: The mean filter estimates $\bar{\mathbf{c}}(n)$ and the steady state value $J(\infty)$ of our performance measure satisfy

$$\begin{aligned} \bar{\mathbf{c}}_t(n) &= \Big(\mathbf{I} - \mu(\mathbf{I} - \mathbf{s}_1\mathbf{s}_1^t)\mathbf{\Sigma}_{\tilde{\mathbf{r}}}\Big)\bar{\mathbf{c}}_t(n-1) \\ \bar{\mathbf{c}}_d(n) &= \Big(\mathbf{I} - \mu(\mathbf{I} - \mathbf{s}_1\mathbf{s}_1^t)\mathbf{\Sigma}_{\tilde{\mathbf{r}}}\rho\Big(\tfrac{a_1}{\|\mathbf{\Sigma}_{\tilde{\mathbf{r}}}^{1/2}\bar{\mathbf{c}}_d(n-1)\|}\Big)\Big)\bar{\mathbf{c}}_d(n-1) \\ J_t(\infty) &\approx \tfrac{\mu}{2}\Big(\textstyle\sum_{i=2}^{K} a_i^2 + (N-1)\sigma^2\Big)\Big(\tfrac{\sigma^2}{a_1^2} + (1 - \tfrac{\alpha}{a_1})^2\Big) \\ J_d(\infty) &\approx \tfrac{\mu}{2}\Big(\textstyle\sum_{i=2}^{K} a_i^2 + (N-1)\sigma^2\Big)\times \\ &\qquad \Big(\tfrac{\sigma^2}{a_1^2} + 4Q(\tfrac{a_1}{\sigma}) - \tfrac{4}{\sqrt{2\pi}}\tfrac{\sigma}{a_1}e^{-\frac{a_1^2}{2\sigma^2}}\Big)\rho^{-1}(\tfrac{a_1}{\sigma}), \end{aligned}$$

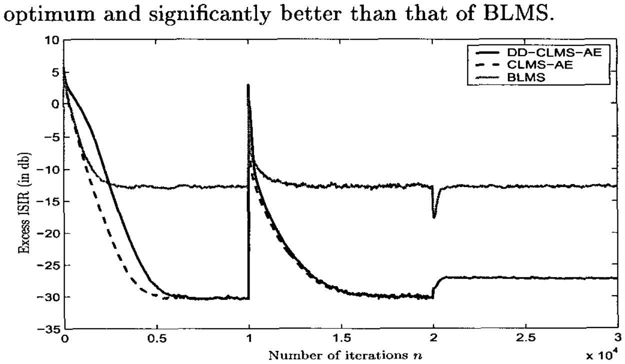
where $\rho(x) = 1 - \sqrt{2/\pi}xe^{-x^2/2}$ and $\mathbf{\Sigma}_{\tilde{\mathbf{r}}}$ is the covariance matrix of the interference plus noise part of the data. For the last two relation we assumed approximately orthogonal signatures.

From the Theorem we can conclude [2] that the best trained version corresponds to $\alpha = a_1$, i.e. CLMS-AI. This is the reason why we analyze the DD version of this case only (i.e. DD-CLMS-AI). Furthermore BLMS ($\alpha = 0$) is better than CLMS ($\alpha = 1$) when $a_1 < 0.5$. Comparing now CLMS-AI to DD-CLMS-AI one concludes that the DD version is very close to the optimum trained version even for moderate values of SNR and at the same time significantly better than BLMS.

The optimum algorithm CLMS-AI and its DD version require knowledge of a_1. One can estimate the amplitude directly from the data $\mathbf{r}(n)$ using the filter estimates $\mathbf{c}$ as follows

$$\begin{aligned} \hat{a}_1(n) &= (1-\nu)\hat{a}_1(n-1) + \nu\mathbf{c}^t(n-1)\mathbf{r}(n)b_1(n) \\ \hat{a}_1(n) &= (1-\nu)\hat{a}_1(n-1) + \nu|\mathbf{c}^t(n-1)\mathbf{r}(n)|, \end{aligned}$$

with the first formula applied to the trained case and the second to the DD. Let us call the resulting algorithms CLMS-AE and DD-CLMS-AE respectively (where AE stands for amplitude estimation). In our figure we present the relative performance of the two algorithms and that of the BLMS of [1]. We have selected $N = 128$ and SNR=20 db; initially there are twenty nine 10 db interferers; at $n = 10000$ five 20 db interferers enter and at 20000 all five 20 db along with five 10 db interferers exit the channel. We observe an initial nonlinear behavior of DD-CLMS-AE; however the algorithm, very quickly, attains the same convergence rate as CLMS-AE and a steady state performance which is indistinguishable from the optimum and significantly better than that of BLMS.



References

[1] M. Honig, et.al. "Blind multiuser detection," *IEEE Trans, Inform. Theory*, vol. IT-41, pp. 944-960, July 1995.

[2] G.V. Moustakides, "Constraint adaptive linear multiuser detection schemes," invited paper, special issue on multiuser detection, *J. VLSI SP*, (under review).

[1]This work was supported by a Collaborative Research Grant from the NATO International Scientific Exchange Programme.

0-7803-7123-2/01/$10.00 ©2001 IEEE