EYE DIAGRAM RECONSTRUCTION USING ASYNCHRONOUS IMPERFECT SAMPLING, APPLICATION TO BER ESTIMATION FOR FIBER-OPTIC COMMUNICATION SYSTEMS *

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ABSTRACT

In this paper we present a new and robust method to construct the eye diagram from asynchronous samples of a digital communication signal. No a priori knowledge of the bit period is needed. The method uses an approach based on periodogram estimation which is suitable even for (highly) under sampled signals. Random shifts due to clock errors are also being corrected. We apply this method to the Bit Error Rate estimation of an optical signal in an experimental setup.

1 INTRODUCTION

The main motivation for this work comes from a problem related to the monitoring of optical networks. In next generations of such networks, optical technology will be more and more used not only for transmission but also for switching (in replacement of present electrical cross-connects) of signals. The objective is to alleviate the bottlenecks due to capacity and cost of electronic solutions and to provide a versatile "transparent" (without opto-electronic conversions) optical network able to carry client signals independently of the various formats of their electrical frame. However, the capacity to monitor the quality of the signal along its path is a required feature to build a manageable network. It is clear that transparent optical networks can exist only if transparent monitoring methods are developed. This is the aim of the technique presented in this paper which successfully measures the main quality indicator of a digital communication system, namely the Bit Error Rate (BER), without accessing neither the electrical frame of the corresponding signal nor even knowing its bit rate.

The received signal can be modeled as

$$x(t) = \sum_{n=-\infty}^{+\infty} a_n\,\phi(t - n\,T_b) + v(t)$$

where $\{a_n\}$ is the transmitted symbol sequence taking only upon two values $a_n \in \{1, \delta\}$ with equal probability where $\delta > 0$; T_b is the bit period; ϕ is a continuous time function with compact support representing the "pulse" shape; and v is some additive (usually Gaussian) noise. Assume that some device can sample the instantaneous optical power, but at a rate significantly lower than the bit rate, and without any synchronization. The question one can pose is what useful information, concerning the quality of the acquired signal, can be extracted from such samples?

If the signal is highly sub-sampled, then traditional Signal Processing techniques fail, and the idea that comes to mind is to use the available samples to build an eye diagram. This is indeed possible without any information on the optical signal, as we will show next. In Section 3, these ideas will applied to the problem of BER estimation.

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2 EYE DIAGRAM RECONSTRUCTION

Given the samples $x_n = x(t_n)$ we would like to reconstruct the eye diagram, that is, the set of pairs $(t_n \bmod T_b, x_n)$, or equivalently $(\frac{t_n}{T_b}, x_n)$. We assume that the sampling times are given by $t_n = t_0 + n\,T_s + w_n$ where T_s is the sampling period (we may have $T_s >> T_b$), and w_n is some random perturbation (usually a random walk) due to clock's lack of accuracy. We also assume that T_s/T_b is not an integer. We show first that we do not need to estimate explicitly T_b, not even the ratio $\frac{T_s}{T_b}$ to construct the eye diagram. Indeed, by denoting $f_b = T_b^{-1}$ and $f_s = T_s^{-1}$, we have the following identities when $w_n = 0$

$$\begin{aligned}\frac{t_n}{T_b} \bmod 1 &= \frac{t_0 + n\,T_s}{T_b} \bmod 1 = (\frac{t_0}{T_b} + n\frac{f_b}{f_s}) \bmod 1 \\ &= (\frac{t_0}{T_b} + n(\frac{f_b}{f_s} - \mathrm{round}(\frac{f_b}{f_s}))) \bmod 1 \\ &= (\varepsilon_b(\nu + n\,\lambda_b)) \bmod 1,\end{aligned}$$

where $\lambda_b = |\frac{f_b}{f_s} - \mathrm{round}(\frac{f_b}{f_s})|$ is the aliased digital version of the bit frequency f_b, ν is the initial time shift, and ε_b is $+1$ or -1. This means that in order to make the eye diagram (up to a time reversal) we only need to know λ_b.

To estimate λ_b, it is natural to think of the periodogram method. Unfortunately the bit frequency does not usually appear clearly on the signal spectrum as we can see in Fig. 1. The idea is to apply a non linear

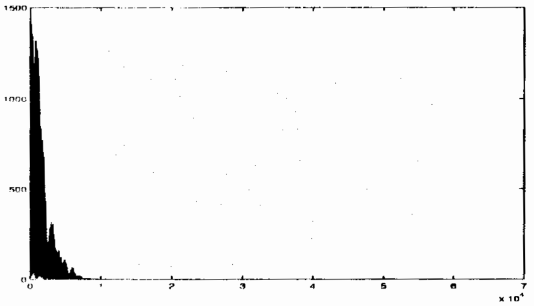

Figure 1: Spectrum of the signal $x(t)$

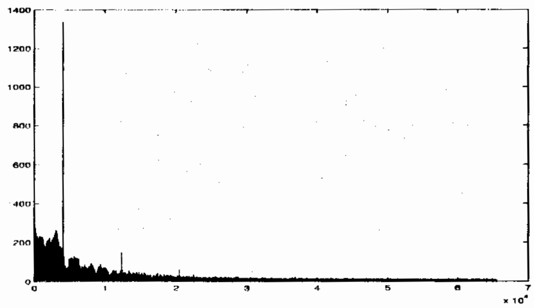

Figure 2: Spectrum of $x(t)^{0.1}$

function to the signal $x(t)$ to force the bit frequency to manifest itself. This can be done by applying a function which emphasizes the transitions between the two levels (corresponding to 0 and 1 in the transmitted sequence). Several functions may be used and give satisfactory results, a possible choice is, for example, $f(x) = |x|^{0.1}$. The resulting spectrum, computed by the periodogram method of the next subsection, plotted in Fig. 2, reveals clearly the desired (aliased) bit frequency λ_b. Other suitable f functions are $f(x) = |x|^5$ or $f(x) = 1_{]\alpha,1]}(|x|)$ for properly selected level α.

2.1 Periodogram

Once function f is selected, we can consider the transformed samples $y_n = f(x_n)$, $n = 0, \ldots, N-1$. By dividing the available samples into M blocks, and assuming for simplicity, $N = ML$ we can define the periodogram of y_n as follows

$$P(e^{j\omega}) = \frac{1}{M} \sum_{k=0}^{M-1} |Y_k(e^{j\omega})|^2,$$

where $Y_k(e^{j\omega})$ is the Fourier transform of the samples in the k-th block, i.e.

$$Y_k(e^{j\omega}) = \sum_{m=0}^{L-1} W_m \, y_{kL+m} \, e^{-jm\omega},$$

with W_m being simply a windowing sequence to attenuate the Gibbs phenomenon.

2.1.1 *Perfect sampling*

Under perfect sampling, that is $t_n = t_0 + n\,T_s$, the modified samples y_n may be written as

$$y_n = \sum_{k=-\infty}^{\infty} A_k \, e^{j\,n\,\omega_b^k} + v_n,$$

where $\omega_b^k = 2\pi\,\lambda_b^k$ with λ_b^k being the aliased version of the harmonic $k\,f_b$, and v_n is the non periodic part. For large M, by virtue of the Law of Large Numbers, $P(e^{j\omega})$ is approximately $\mathbb{E}[|Y(e^{j\omega})|^2]$. Taking for simplicity $W_n = 1$ we can then write

$$\begin{aligned} \mathbb{E}[|Y(e^{j\omega})|^2] &= \left| \sum_{k=-\infty}^{\infty} A_k \frac{1 - e^{j\,L\,(\omega_b^k - \omega)}}{1 - e^{j\,(\omega_b^k - \omega)}} \right|^2 \\ &\quad + \mathbb{E}\left[\left| \sum_{n=0}^{L-1} v_n \, e^{-j\,n\,\omega} \right|^2 \right]. \end{aligned}$$

In the last equation, the first term in the right hand side will be dominant for the largest component A_k and for $\omega = \omega_b^k$. Let us, for simplicity, assume that v_n is modeled as white noise. Then the first term is proportional to L^2 and the second only to L. Therefore for sufficiently large block size L we will be able to distinguish clearly λ_b.

2.1.2 *Imperfect sampling*

In this case, $t_n = t_0 + n\,T_s + w_n$, and we can write y_n as

$$y_n = \sum_{k=-\infty}^{\infty} A_k \, e^{j\,2k\pi\,f_b(nT_s + w_n)} + v_n.$$

Let us define $\Omega_b = 2\pi\,f_b$ and $\tilde{A}_k(n) = A_k\,e^{jk\omega_b w_n}$. Then

$$y_n = \sum_{k=-\infty}^{\infty} \tilde{A}_k(n)\, e^{jn\omega_b^k} + v_n,$$

which shows that the imperfect sampler introduces random phases at frequencies ω_b^k. We concentrate on $\omega = \omega_b = \omega_b^1$. Assuming L sufficiently large, so that contributions from other harmonics are negligible, we can write

$$\begin{aligned} Y(e^{j\omega_b}) &= \sum_{n=0}^{L-1} y_n \, e^{-jn\omega_b} \\ &\simeq A_1 \sum_{n=0}^{L-1} e^{j\Omega_b w_n} + \sum_{n=0}^{L-1} v_n \, e^{-jn\omega_b}. \end{aligned}$$

Here the first term is due to the periodic part of the signal, but it is no longer equal to $A_1 L$ as it was for the perfect sampler ($w_n = 0$). Considering the first sum as "signal" and the second one "noise" we can define the SNR as

$$\mathrm{SNR}(L) = \frac{|A_1|^2}{\sigma_v^2} \frac{1}{L} \mathbb{E}\left[\left| \sum_{n=0}^{L-1} e^{j\Omega_b w_n} \right|^2 \right].$$

It is more convenient now, to define the SNR gain (SNRG) as the improvement obtained by using blocks of size L instead of size 1, namely

$$\mathrm{SNRG}(L) = \frac{\mathrm{SNR}(L)}{\mathrm{SNR}(1)} = \frac{1}{L}\mathbb{E}\left[\left|\sum_{n=0}^{L-1} e^{j\Omega_b w_n}\right|^2\right].$$

If we assume that w_n is Gaussian random walk (which is the most well accepted model in practice), then the steps $w_n - w_k$ (for $n > k$) are centered Gaussian variables with variance $(n-k)\sigma_w^2$. Let us denote $\beta = e^{-\sigma_w^2\Omega_b^2}$ then we can show that

$$\begin{aligned}\mathbb{E}\left[\left|\sum_{n=0}^{L-1} e^{j\Omega_b w_n}\right|^2\right] &= L + 2\sum_{n>k}\mathrm{Re}\left(\mathbb{E}[e^{j\Omega_b(w_n-w_k)}]\right)\\ &= L + 2\sum_{n=1}^{L-1} n\, e^{(L-n)\sigma_w^2\Omega_b^2}\\ &= L + 2\beta\frac{(L-1) - L\beta - \beta^L}{(1-\beta)^2}.\end{aligned}$$

And the SNR gain becomes

$$\mathrm{SNRG}(L) = \frac{1+\beta}{1-\beta} + 2\beta\frac{1-\beta^L}{L(1-\beta)^2},$$

which tends to $\frac{1+\beta}{1-\beta}$ as L approaches ∞. We therefore conclude that the gain is larger than 1, however the improvement saturates. This suggests that, in practice for fixed sample size, we need to tune the selection of L. The periodogram of a sequence sampled with imperfect sample is presented in Fig. 3. The estimate of the desired frequency, as we can see, is still extremely accurate.

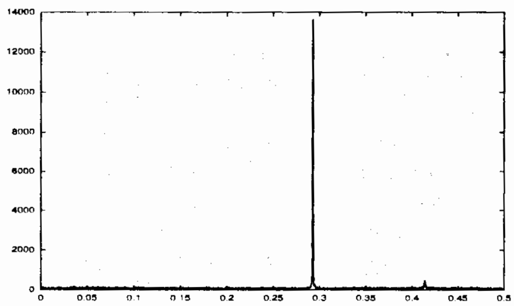

Figure 3: Example of periodogram, $M = 20$, $L = 2316$

2.2 Correcting phase shifts

When the samples are not accurate because of clock jitter (random walk case) we have seen that it is still possible to obtain accurate estimates of λ_b. However, this is not sufficient for forming the eye diagram. Indeed, even if we had available the exact λ_b the eye diagram would had been as in Fig. 4 because of its sensitivity to clock errors. Fortunately it is possible to overcome this problem successfully. The key idea we rely on is the following, if T is any period then for any time instant t we have

$$\frac{t}{T} \bmod 1 = \frac{\arg(e^{j\Omega t})}{2\pi}$$

with $\Omega = \frac{2\pi}{T}$. From this we conclude that finding the position of a sample in the eye diagram can be viewed as a synchronization to a reference exponential $e^{j\omega_b t}$. We already know that the modified samples $\{y_n\}$ have a strong spectral component at ω_b, so it is natural to use them to synchronize the sequence $\{x_n\}$. Since clock errors are of random walk type, this means that locally the phase shifts are small, but they accumulate with time. Therefore if we select a time window of sufficiently small length say $2K+1$ and compute

$$Y_n = \sum_{k=-K}^{K} y_{n+k} W_k\, e^{-jk\omega_b},$$

where W_k is again a windowing sequence, then the resulting eye diagram is given by the pairs

$$\{(\tau_n, x_n)\},\ \text{where } \tau_n = \frac{\arg(Y_n)}{2\pi}.$$

This method turned out to be very successful, as we can see from Figs. 4 and 5. It is also worth mentioning that the transformation yielding the sequence Y_n can be viewed as an FIR filter of constant coefficients

$$W_K e^{jK\omega_b}, \ldots, W_1 e^{j\omega_b}, 1, W_1 e^{j\omega_b}, \ldots, W_K e^{-jK\omega_b}.$$

applied to the transformed data y_n .

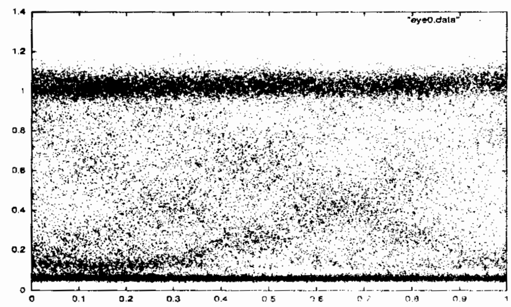

Figure 4: Eye diagram without phase shift correction

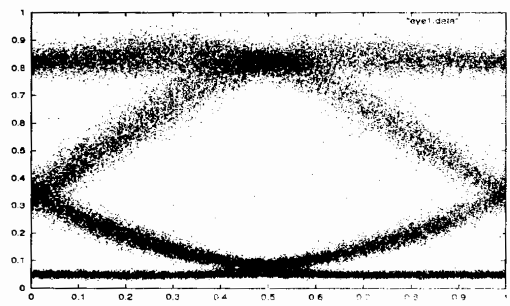

Figure 5: Eye diagram with phase shift correction

3 APPLICATION TO BER ESTIMATION

As stated in the introduction, our motivation for this work was the monitoring of optical networks. The most important measure of quality of service in optical fibers and communication systems in general is definitely the BER. The detector consists mainly of a photodiode, followed by a decision circuit comparing the received power to some threshold, and of a synchronization device. We make an error each time a "0" is above or a a "1" below the threshold.

If we want to estimate the BER at any point in the network in a transparent way, that is, with minimum knowledge and dependence on the actual signal form, we could use samples of the instantaneous optical power. The samples can be acquired using a device similar to the sampling head of a modern digital oscilloscope. When the samples are not synchronized, some authors propose to estimate the BER by using histograms (see [2], [5], [6]). To be able to obtain sufficiently accurate estimates of the BER, the histogram needs to be formed using samples that correspond to time instances close to decision (the region where the eye is wide open). Selecting such points from asynchronously acquired data is not an easy task. To avoid this problem it is possible to use a synchronization device, construct the eye diagram, and then estimate the probability density function at any time instant in the eye [4]; or alternatively use a Gaussian model setup [3].

It turns out that synchronization can be performed by means of software, without any special knowledge of the signal. The method presented in the previous section was successfully applied in this framework, to form the eye diagram of an optical signal. Experiments were performed at Alcatel, at a bit rate of 10 Gbits/s and a sampling frequency of 50 kHz, corresponding to an under–sampling factor of 200 000. After the reconstruction of the eye diagram, one selects the points correspond to times close to the decision instant, i.e. where the eye is most open. BER is then estimated from this data set, using a Gaussian mixture model. Specifically we consider that samples are i.i.d. with a probability density being a mixture of eight Gaussians of the form

$$p(x) = \frac{1}{8}\sum_{i=1}^{8}\frac{1}{\sqrt{2\pi\sigma_i^2}}\exp\left(\frac{1}{2\sigma_i^2}(x-m_i)^2\right).$$

The means $\{m_i\}$ and variances $\{\sigma_i^2\}$ can be estimated using the EM algorithm [1]. The selection of eight Gaussians was necessary to account for the inter-symbol-interference of neighboring bits. Once the sixteen parameters of the model are estimated, we can compute the optimal decision threshold and then the BER using Gaussian statistics.

The results of our method were compared against the true experimental BER, resulting from a "BER–meter" device that compares the output of the detector to the correct transmitted symbol sequence. Figure 6 depicts the comparison of the experimental BER versus the estimated one. We observe a satisfactory agreement between the two curves.

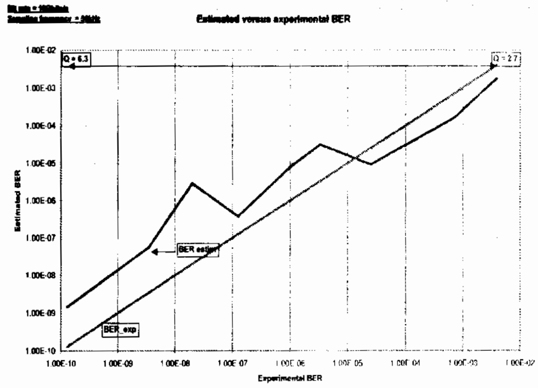

Figure 6: Comparison of estimated and experimental BER

4 CONCLUSION

We have developed and experimentally validated a method for reconstructing eye diagrams and estimating BER of an optical communication system. The proposed method uses asynchronous and imperfect sampling of the optical signal and required no knowledge of the bit and sampling rates. A large under-sampling factor is tolerated thanks to an algorithm particularly robust against signal and sampling clock jitter. The application of our results in future transparent optical networks would confer them a crucial feature, namely the ability to reliably monitor the quality of the signal independently of its electrical frame format and/or bit rate.

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