Blind Channel Estimation for Downlink CDMA Systems

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Abstract— **The problem of channel estimation in code-division multiple-access (CDMA) systems is considered. Using only the spreading code of the user of interest, a technique is proposed to identify the impulse response of the multipath channel from the received data sequence. While existing blind methods suffer from high computational complexity and sensitivity to accurate knowledge of the noise subspace rank, the proposed method overcomes both problems. In particular we estimate the noise subspace by a simple matrix power that is computationally efficient and requires no knowledge of the noise subspace rank. Once an estimate of the noise subspace is available the channel impulse response can be directly identified through a small size (order of the channel) SVD or a least squares approach. Extensive simulations demonstrate similar performance of our method as compared to the existing schemes but at a considerably lower computational cost.**

I. Introduction

CODE-DIVISION multiple-access (CDMA) implemented with direct-sequence spread spectrum (DS/SS) constitutes one of the most important emerging technologies in wireless communications. It is well known that CDMA has been selected as the base for the 3-rd generation mobile telephone systems.

In a CDMA system a signature waveform is assigned to each user thus allowing users to simultaneously transmit in time and occupy the same frequency band. This important advantage is at the same time the main source of performance degradation since to each user all other users play the role of interference. At the receiver end (the mobile unit) each user receives the information transmitted by the base station and needs to detect the information that is destined to him by screening out all interfering users. Several popular multiuser detection structures are reported in detail in [4]. All detectors, in order to be practically implementable, require at least knowledge of the signature waveform of the user of interest. Assuming that such an information is practically available, is in fact quite realistic.

In CDMA, when there is multipath, the effective signature signals are no longer the signature waveforms but rather the convolution of these signals with the unknown channel impulse response. This combined waveform is also known as *composite signature*. It is thus clear that if we like to apply the detection structures of [4] introduced for the non-dispersive channel case we need to know (or efficiently estimate) the channel impulse response. Furthermore, it is only natural to express a strong interest towards blind estimation methods, since they do not require transmission of any training sequences.

In the literature there is a very limited number of results reported on the problem of blind channel estimation for CDMA. In [1] and [2] the channel impulse response is recovered with the help of a two-step procedure. The first step consists in the application of an SVD onto a large matrix in order to obtain a base for the noise subspace of the received signal. The second step involves either an SVD [1] or a QR decomposition [2] applied to a small sized matrix (of the order of the channel), that yields the final impulse response estimate. Furthermore in [2], by applying successive QR decompositions, it is possible to perform timing synchronization, with respect to the user of interest.

The main drawback of both methods consists in their need for performing SVD on a large matrix in order to obtain a basis for the noise subspace. This practically limits the use of both methods to small spreading codes, with codes, for example, of size $N = 128$ being practically inaccessible. We should also mention the fact that both approaches are very sensitive to the correct knowledge of the noise subspace rank. This parameter is not constant since it changes every time users are entering or exiting the channel. It turns out that erroneous rank estimates lead to drastic performance degradation even in the short code case.

An alternative and recently proposed approach [3], based on higher order cumulants, overcomes both drawbacks of the previous two schemes. This method however, requires an iterative implementation, that suffers from an exceedingly slow convergence even for very small codes. Furthermore its success relies on the Gaussian noise assumption and in particular that higher order cumulants of Gaussian random variables are zero.

The idea we propose here seems to overcome all drawbacks reported above for the existing approaches. More specifically our method follows the main lines of [1], [2] with a very essential difference. We replace the first large SVD step by the computation of a matrix power. Although, in theory, the power method attains the performance of SVD only in the limit, as the power tends to infinity, in practice we do not need to go beyond the third one. In fact, most of the time, even the first and second power exhibits excellent performance. Moreover the power method does not require any knowledge of the noise subspace rank thus its robustness, with respect to this parameter, is guaranteed.

As far as the second step is concerned, we can select between a small sized SVD [1], a QR decomposition [2], or finally a simple least squares (LS) approach (proposed here). As far as the latter approach is concerned, it should be mentioned that, it also lends itself to the definition of an efficient scheme

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for resolving the timing synchronization problem. Extensive simulations demonstrate rapid convergence of our method and performance that is comparable to [1], [2] but at a significantly lower computational cost. Processing codes of size 128 or 256 poses no particular problem, not to mention the fact that it is also possible to develop adaptive algorithms, for each step of our method, with computational complexity that can be handled with todays real time processing platforms.

The rest of the paper is organized as follows. In Section II, we introduce the signal model for synchronous CDMA. In Section III we present our main results. In Section IV we perform a number of simulation comparisons between the proposed and the existing methods. Finally Section V contains our conclusions.

II. Signal Model

We are focusing on the downlink scenario where all users are synchronized. Consider a K-user DS-CDMA system with identical chip waveforms and signaling antipodally through a multipath channel in the presence of additive white noise (AWN). Although the signals appearing in CDMA system are continuous in time, the system we are interested in, can be adequately modeled by an equivalent discrete time system [4]. Specifically, no information is lost if we limit ourselves to the discrete time output of a chip matched filter applied to the received analog signal [4]. Let N be the processing gain of the code and L the length of the channel impulse response. Without loss of generality, throughout this article, we will assume that the user of interest is User-1; we will also assume that the initial delay is known and therefore we have exact synchronization with the user. The last assumption is not very restrictive since, as was mentioned in the Introduction, our method can be easily extended to include estimates of this initial delay.

Let $z(n)$ be the signal transmitted by the base station, then

$$z(n) = \sum_{i=1}^{K} \sum_{k=-\infty}^{\infty} a_i s_i(n-kN) b_i(k), \qquad (1)$$

where a_i is the amplitude of User-i, $b_i(k)$ his corresponding bit sequence and $\mathbf{s}_i = [s_i(0)\, s_i(1) \cdots s_i(N-1)]^t$ his length N normalized (i.e. $\|\mathbf{s}_i\| = 1$) signature waveform.

Signal $z(n)$ propagates through a multipath AWN channel with impulse response $\mathbf{f} = [f(0) \ \cdots \ f(L-1)]^t$ therefore, at the receiver end, the received signal $y(n)$ takes the form

$$\begin{aligned} y(n) &= z(n) \star f(n) + \sigma w(n) \\ &= \sum_{i=1}^{K} \sum_{k=-\infty}^{\infty} a_i \tilde{s}_i(n-kN) b_i(k) + \sigma w(n), \end{aligned} \qquad (2)$$

where $\star$ denotes convolution; $\tilde{s}_i(n) = s_i(n) \star f(n)$ is the convolution between the initial signature vector $\mathbf{s}_i$ and the channel impulse response $\mathbf{f}$ (i.e. the composite signature of User-i); and finally σ^2 denotes the power of the AWN.

For the presentation of our method it is more convenient to express the received signal in blocks of data. In particular we are interested in blocks of size $mN + L - 1$, where m is a positive integer. Consequently let us consider the following block

$$\mathbf{r}_m(n) = [y(nN) \ \cdots y((n-m)N - L + 2)]^t \qquad (3)$$

which is assumed synchronized with the user of interest. Notice that due to synchronization, inside each block $\mathbf{r}_m(n)$ exist m entire copies of the composite signature of the user of interest. To illustrate this fact and also specify in more detail the different components of the received signal vector, let us analyze the case $m = 2$. Vector $\mathbf{r}_2(n)$ can be decomposed as follows

$$\begin{aligned} \mathbf{r}_2(n) &= \begin{bmatrix} \tilde{\mathbf{s}}_1 \\ \mathbf{0}_{N\times 1} \end{bmatrix} a_1 b_1(n) + \begin{bmatrix} \mathbf{0}_{N\times 1} \\ \tilde{\mathbf{s}}_1 \end{bmatrix} a_1 b_1(n-1) \\ &+ \sum_{i=2}^{K} \left(\begin{bmatrix} \tilde{\mathbf{s}}_i \\ \mathbf{0}_{N\times 1} \end{bmatrix} a_i b_i(n) + \begin{bmatrix} \mathbf{0}_{N\times 1} \\ \tilde{\mathbf{s}}_i \end{bmatrix} a_i b_i(n-1) \right) \\ &+ \text{ISI} + \sigma \mathbf{w}_2(n), \end{aligned} \qquad (4)$$

where $\tilde{\mathbf{s}}_i = [\tilde{s}_i(0) \cdots \tilde{s}_i(N+L-2)]^t$ is the composite signature of User-i. We observe in (4) that the first two terms involve the entire composite signature of the user of interest; the next sum includes the multi-access interference, that is, terms similar to the first two, but corresponding to interfering users (recall that all user are synchronized); then follows the ISI part that includes the inter-symbol interference of all users and finally the last term is the AWN vector. The ISI part, as is the case with the other two parts, involves a sum of terms that are of the form $\mathbf{d}_k b_j(n-i)$ where $\mathbf{d}_k$ suitable vectors and $b_j(n)$ are binary data. It should be noted that all terms in (4), except the noise term, involve binary data that are mutually independent and also independent of the noise vector.

One final point, we should mention, before proceeding with the presentation of our results is the fact that the composite signature can be written as

$$\tilde{\mathbf{s}}_i = \mathbf{S}_i \mathbf{f} \qquad (5)$$

where $\mathbf{S}_i$ is a convolution matrix of dimensions $(N+L-1) \times L$, corresponding to the signature of User-i and defined as

$$\mathbf{S}_i = \begin{bmatrix} s_i(0) & 0 & \cdots & 0 \\ \vdots & s_i(0) & \ddots & \vdots \\ s_i(N-1) & \vdots & \ddots & 0 \\ 0 & s_i(N-1) & \ddots & s_i(0) \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & s_i(N-1) \end{bmatrix}. \qquad (6)$$

III. Main Results

Following the main lines of the method in [1], [2], our channel estimation scheme involves two main steps. The first consists in estimating a basis for the noise subspace of the received signal or, equivalently, an alternative quantity that is more suitable for the channel estimation problem. The second step, once the information regarding the noise subspace is available, consists in estimating the final channel impulse response.

Let us first deal with the decomposition of the received data into the signal and noise subspace. Consider the autocorrelation matrix $\mathbf{R}_{\mathbf{r}_m}$ of the received data vector $\mathbf{r}_m(n)$ defined in (3)

$$\mathbf{R}_{\mathbf{r}_m} \stackrel{\triangle}{=} E\{\mathbf{r}_m(n)\mathbf{r}_m^t(n)\} = \mathbf{Q} + \sigma^2 \mathbf{I} \qquad (7)$$

where $\mathbf{Q}$ is a $(mN+L-1)\times(mN+L-1)$ matrix of the form

$$\mathbf{Q}=\sum_i \mathbf{d}_i\mathbf{d}_i^t, \tag{8}$$

where $\mathbf{d}_i$ are the vectors coming from (4) and are due either to the user of interest, the MAI or the ISI part.

Consider now the SVD of the matrix $\mathbf{Q}$

$$\mathbf{Q}=[\mathbf{U}_s \quad \mathbf{U}_n]\begin{bmatrix}\mathbf{\Lambda}_s & \mathbf{0}\\ \mathbf{0} & \mathbf{0}\end{bmatrix}[\mathbf{U}_s \quad \mathbf{U}_n]^t. \tag{9}$$

This leads to the following SVD of the autocorrelation matrix

$$\mathbf{R}_{\mathbf{r}_m}=[\mathbf{U}_s \quad \mathbf{U}_n]\begin{bmatrix}\mathbf{\Lambda}_s+\sigma^2\mathbf{I} & \mathbf{0}\\ \mathbf{0} & \sigma^2\mathbf{I}\end{bmatrix}[\mathbf{U}_s \quad \mathbf{U}_n]^t, \tag{10}$$

where $\mathbf{U}_s, \mathbf{U}_n$ are orthonormal bases of the signal and noise subspace respectively. We can then see that the singular values corresponding to the noise subspace are all equal to σ^2 and are the smallest ones. Based on the decomposition in (10) we have the following results.

Lemma 1: Let $\mathbf{R}_{\mathbf{r}_m}$ be the autocorrelation matrix defined in (7) of the received data vector $\mathbf{r}_m(n)$. If $\mathbf{U}_n$ is the orthonormal base of the noise subspace appearing in (10), then we have the following limit

$$\lim_{k\to\infty}(\sigma^2\mathbf{R}_{\mathbf{r}_m}^{-1})^k=\mathbf{U}_n\mathbf{U}_n^t. \tag{11}$$

Remark: Since $\mathbf{U}_n^t\mathbf{U}_s=0$, we conclude that for any vector $\mathbf{s}$ belonging to the signal subspace we have

$$\mathbf{U}_n^t\mathbf{s}=0. \tag{12}$$

In particular this is true for all vectors $\mathbf{d}_i$ appearing in (8). Using the above orthogonality property we can prove the following lemma.

Lemma 2: Suppose, for the vectors $\mathbf{d}_i$ composing the matrix $\mathbf{Q}$ in (8), a number l of them can be put under the form $\mathbf{d}_i=\mathbf{F}_i\mathbf{f},\ i=1,\ldots,l$, where $\mathbf{F}_i$ known matrices, then

$$\lim_{k\to\infty}\mathbf{f}^t\mathbf{F}^t\big(\sigma^2\mathbf{R}_{\mathbf{r}_m}^{-1}\big)^k\mathbf{F}\mathbf{f}=\mathbf{f}^t\mathbf{F}^t\mathbf{U}_n\mathbf{U}_n^t\mathbf{F}\mathbf{f}=0, \tag{13}$$

whith $\mathbf{F}=\sum_i^l\mathbf{F}_i$.

We have now sufficient information to proceed with the channel estimation method.

A. Key Ideas

If we consider the inverse of the autocorrelation matrix raised to a *finite* power k, then (13) suggests the following two estimates for $\mathbf{f}$.

First, the channel impulse response can be recovered from the following minimization problem

$$\hat{\mathbf{f}}=\arg\min_{\mathbf{f}}\mathbf{f}^t\mathbf{F}^t\big(\sigma^{2k}\mathbf{R}_{\mathbf{r}_m}^{-k}\big)\mathbf{F}\mathbf{f}=\arg\min_{\mathbf{f}}\mathbf{f}^t\mathbf{F}^t\mathbf{R}_{\mathbf{r}_m}^{-k}\mathbf{F}\mathbf{f}; \tag{14}$$

subject to $\|\mathbf{f}\|=1$. In other words we obtain $\mathbf{f}$ as the singular vector corresponding to the smallest singular value of the matrix $\mathbf{F}^t\mathbf{R}_{\mathbf{r}_m}^{-k}\mathbf{F}$.

The second method relies on a constrained LS minimization problem, specifically

$$\hat{\mathbf{f}}_i=\arg\min_{\mathbf{f}}\mathbf{f}^t\mathbf{F}^t\mathbf{R}_{\mathbf{r}_m}^{-k}\mathbf{F}\mathbf{f};\ \ \mathbf{f}^t\mathbf{e}_i=1, \tag{15}$$

where $\mathbf{e}_i, i=1,\ldots,M$, are prespecified vectors. The final candidate vector is the one that is closest to the SVD solution, that is, $\hat{\mathbf{f}}=\hat{\mathbf{f}}_{i_o}$, where i_o is

$$i_o=\arg\min_i\frac{\hat{\mathbf{f}}_i^t\mathbf{F}^t\mathbf{R}_{\mathbf{r}_m}^{-k}\mathbf{F}\hat{\mathbf{f}}_i}{\|\hat{\mathbf{f}}_i\|^2}. \tag{16}$$

We should mention that in the limiting case $k\to\infty$, a single vector $\mathbf{e}_i$ would suffice, since it could make the numerator in (16) exactly zero. A very important observation in our approach is the fact that the product $\mathbf{U}_n\mathbf{U}_n^t$ is approximated by a power of the autocorrelation matrix, without needing any knowledge of the noise subspace rank. This should be compared to the large SVD applied on $\mathbf{R}_{\mathbf{r}_m}$ in [1], [2] where for the determination of $\mathbf{U}_n$, the knowledge of this parameter (or a reliable estimate) is indispensable.

B. Channel Estimation Method

After having introduced the mathematical background on which our method relies, let us now proceed with the presentation of the actual blind channel estimation scheme.

Previously we assumed available the data autocorrelation matrix, knowledge that could lead to exact estimates of the signal and noise subspace and consequently to exact estimates of the channel impulse response. If we now consider the realistic situation where only received data and the signature of the user of interest are available, one should proceed as follows.

The first step consists in estimating the data autocorrelation matrix by averaging over the received blocks, i.e.

$$\hat{\mathbf{R}}_{\mathbf{r}_m}=\frac{1}{M}\sum_{n=1}^{M}\mathbf{r}_m(n)\mathbf{r}_m^t(n). \tag{17}$$

In order now to be able to apply either (14) or (15,16) we need to specify a *known* matrix $\mathbf{F}$ such that $\mathbf{F}\mathbf{f}$ is in the signal subspace. It turns out that this problem is not particularly difficult. Let us consider again the case $m=2$, then from (4) we have that the vectors $[\tilde{\mathbf{s}}_1^t\ \mathbf{0}]^t$ and $[\mathbf{0}\ \tilde{\mathbf{s}}_1^t]^t$ belong to the signal subspace and consequently the same holds for their sum. Because of (5) we can then conclude that for the case $m=2$ we can select

$$\mathbf{F}=\begin{bmatrix}\mathbf{S}_1\\ \mathbf{0}_{N\times L}\end{bmatrix}+\begin{bmatrix}\mathbf{0}_{N\times L}\\ \mathbf{S}_1\end{bmatrix}, \tag{18}$$

with the generalization to any m being straightforward. Since the signature of the user of interest is considered known, matrix $\mathbf{F}$ defined in (18) is known as well.

The estimation problem in (14) takes now the form

$$\hat{\mathbf{f}}=\arg\min_{\mathbf{f}}\mathbf{f}^t\mathbf{F}^t\hat{\mathbf{R}}_{\mathbf{r}_m}^{-k}\mathbf{F}\mathbf{f};\quad \|\mathbf{f}\|=\mathbf{1}, \tag{19}$$

in other words, $\hat{\mathbf{f}}$ is the singular vector corresponding to the smallest singular value of the matrix $\mathbf{F}^t\hat{\mathbf{R}}_{\mathbf{r}_m}^{-k}\mathbf{F}$. We have to mention that the minimization problem in (19) has been already proposed in [5] only for the special case of $k=1$; without providing any theoretical support as to why this scheme could work.

For the estimation problem in (15,16) we first need to specify a collection of vectors $\mathbf{e}_i$. We propose the vectors

$\mathbf{e}_i = [0 \cdots 0\,1\,0 \cdots 0]^t,\ i = 1, \ldots, L$, where the unity is in the i-th position. The minimization in (15) then becomes

$$\hat{\mathbf{f}}_i = \arg\min_{\mathbf{f}} \mathbf{f}^t \mathbf{F}^t \hat{\mathbf{R}}_{\mathbf{r}_m}^{-k} \mathbf{F}\mathbf{f};\ \mathbf{f}^t \mathbf{e}_i = 1, \tag{20}$$

with solution

$$\hat{\mathbf{f}}_i = \frac{(\mathbf{F}^t \hat{\mathbf{R}}_{\mathbf{r}_m}^{-k} \mathbf{F})^{-1} \mathbf{e}_i}{\mathbf{e}_i^t (\mathbf{F}^t \hat{\mathbf{R}}_{\mathbf{r}_m}^{-k} \mathbf{F})^{-1} \mathbf{e}_i}. \tag{21}$$

The final estimate is $\hat{\mathbf{f}} = \hat{\mathbf{f}}_{i_o}$, where

$$i_o = \arg\min_i \frac{1}{\mathbf{e}_i^t (\mathbf{F}^t \hat{\mathbf{R}}_{\mathbf{r}_m}^{-k} \mathbf{F})^{-1} \mathbf{e}_i} \tag{22}$$

$$= \arg\max_i \mathbf{e}_i^t (\mathbf{F}^t \hat{\mathbf{R}}_{\mathbf{r}_m}^{-k} \mathbf{F})^{-1} \mathbf{e}_i \tag{23}$$

It is more convenient to view the solution of the least squares problem as first computing the inverse matrix $(\mathbf{F}^t \hat{\mathbf{R}}_{\mathbf{r}_m}^{-k} \mathbf{F})^{-1}$ then selecting its largest diagonal element, whose position identifies i_o. The final channel estimate is the i_o-th column of $(\mathbf{F}^t \hat{\mathbf{R}}_{\mathbf{r}_m}^{-k} \mathbf{F})^{-1}$ normalized to unit norm.

It is worth mentioning that with a simple modification of our algorithmic scheme it is possible to make very accurate estimates of the initial delay and therefore achieve reliable synchronization without significant increase in the computational complexity. Furthermore all parts of our method allow for adaptive implementations with very low computational complexity. Due however to lack of space, we are not presenting these results.

Computing the product $\mathbf{U}_n \mathbf{U}_n^t$ with the power method is computationally very efficient. In [1], [2], where the matrix $\mathbf{U}_n$ is estimated instead, there is also the need to reliably identify the rank of the noise subspace. Information theoretic criteria such as the Akaike's information criterion (AIC) or the minimum description length (MDL) criterion are employed (for details see [1]). In case of incorrect rank estimation we can have a considerable performance degradation as we will find out next. This should be compared against the simple power method proposed here were such estimates are not necessary.

IV. Simulations - Comparisons

In this section, we provide several simulation results to demonstrate the performance of the blind channel estimation scheme developed in this paper. Randomly generated sequences of length $N = 16$ are used as spreading codes. Once generated, the codes are kept constant for the whole simulation set. The number of blocks that are processed together is $m = 3$ and the load of the system is $K = 10$ users. User-1 is assumed to be the user of interest having unit power. All other users are assumed to have the same power level, which is 20 db higher than User-1. Moreover, all graphs presented in the figures are the result of an average over 100 independent runs.

Fig. 1 depicts the mean square channel estimation error, as a function of the received bits, of the SVD and LS versions of (19) and (23) respectively, and the method proposed in [1]. Here the desired user's SNR is 20 db, and the multipath channel has order $L = 3$. The graphs start from bit 50, in order for the matrix $\hat{\mathbf{R}}_{\mathbf{r}_3}$ to become full rank. [1]. We observe that with power $k = 2$ the performance of both versions, LS and SVD, follow very closely the performance of the computationally demanding method of [1]. Undoubtedly, we can also remark the excellent performance of both versions even with power $k = 1$, attaining very quickly less than -35 db. Furthermore, with power $k = 2$ we reach maximum performance for both versions which is only 2 db inferior to the performance of [1]. No significant additional gain is observed by employing higher values of k.

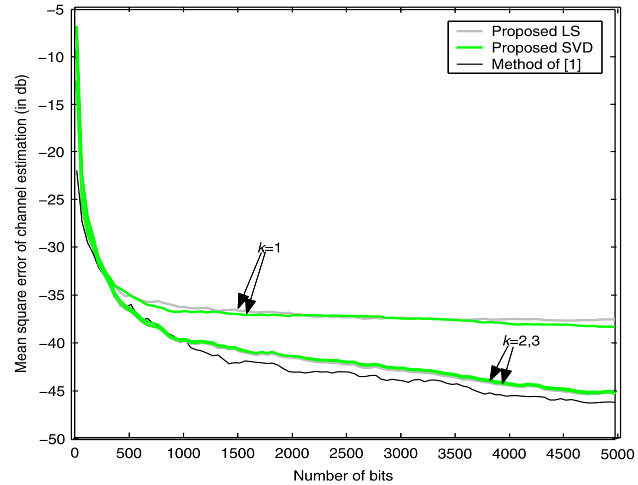

Fig. 1. Performance of proposed channel estimation schemes versus the method of [1]; noise power 20 db.

We next consider the same scenario but with a significantly lower SNR. In particular we consider the desired user's SNR to be 10 db, while all other parameters remain the same as in the previous simulation. Fig. 2 presents the performance of the proposed SVD and LS version versus the method of [1]. We observe again that the two versions are very close to each other, and that in this high noise environment, power $k = 3$ reaches performance that differs from [1] only by 2 db.

The next figure depicts an example of performance degradation of the method in [1] when there is an underestimation of the signal subspace rank just by one unity. The degraded performance corresponds to the dashed line, whereas the one

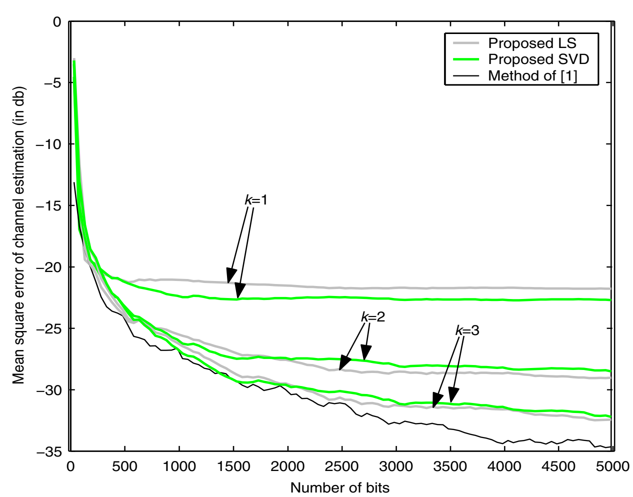

Fig. 2. Performance of proposed channel estimation schemes versus the method of [1]; noise power 10 db.

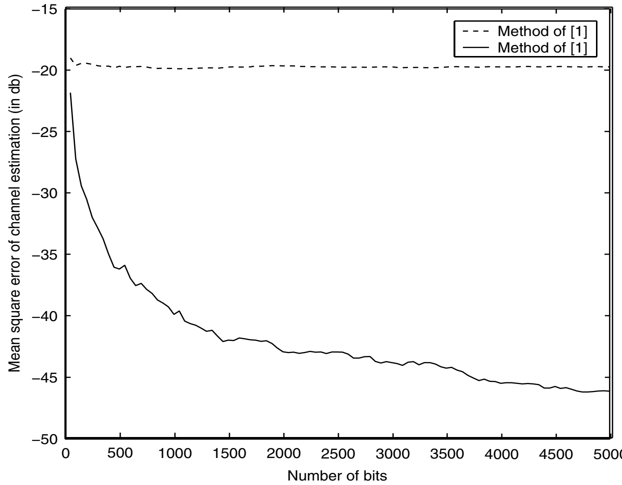

Fig. 3. Performance of method of [1] with and without correct subspace rank estimation; noise power 20 db.

with the correct rank to the solid one. The parameters are exactly as in the first simulation that is, $N = 16$, $K = 10$, $m = 3$, SNR=20 db and $L = 3$. We observe a serious performance degradation of the method of [1] (more than 25 db) when we make an incorrect estimate of the signal subspace rank. It should also be noted that if instead of $m = 3$ we use $m = 1$ then [1], under rank underestimation, attains at best -5 db whereas the proposed versions have similar performance as $m = 3$.

In the last simulation we consider signature codes of length $N = 128$. The total number of users is now $K = 80$, with 29 of them having the same power as User-1; 30 users being 10 db stronger; and the remaining 20 users being 20 db stronger than the user of interest. The channel consists of $L = 10$ coefficients and the SNR level is set to 20 db; finally the number of blocks processed together is, as before, $m = 3$. The performance of the LS and SVD version is presented in Fig. 4, with the graphs starting from bit 400 in order for the initial autocorelation matrix to be of full rank. Again SVD exhibits an excellent performance even from the first power $k = 1$, while both methods attain maximum (in fact indistinguishable) performance for $k = 2$. The method of [1] was not possible to present here due to its excessively long execution time, which was 16 times larger than our most demanding version (SVD with $k = 3$). We recall that both proposed versions (LS and SVD) can be easily implemented adaptively using LMS or RLS type recursions with low computational complexity. Furthermore these realization are capable of following changes in the channel and in the signaling conditions (as changes in the number of users). Adaptive realizations for the method of [1] are also possible through subspace tracking algorithms however such approaches are even more sensitive to the correct knowledge of the subspace rank than off line techniques.

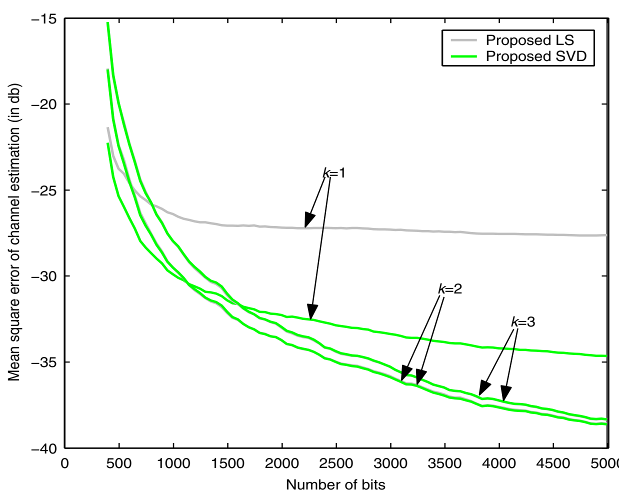

Fig. 4. Performance of proposed channel estimation schemes for codes of length $N = 128$; noise power 20 db.

It is clear from our simulations that in the majority of cases and even under extreme signaling conditions, it suffices to consider a power of $k = 3$. Moreover for high SNR, of the order of 20 db, both proposed versions attain maximum performance with $k = 2$. Finally our SVD version, even with power $k = 1$ yields excellent results.

V. Conclusion

In this paper, we have considered the problem of blind channel estimation for DS-CDMA signals in multipath AWGN channels. A two step channel estimation method was proposed following the same lines of the scheme considered in [1], [2]. The novelty of our approach consists in replacing the first step of [1], [2] that involves a large and computationally demanding singular value decomposition with a simple and computationally efficient matrix power. Regarding the first step it should also be mentioned that in our approach there is no need to have an a-priori knowledge or an accurate estimate of the signal/noise subspace rank, as is the case in [1], [2], where such knowledge is indispensable.

In the second step, except the usual small sized SVD proposed in [1], or the QR decomposition of [2], we introduced a computationally efficient least squares scheme that can also be used for accurate synchronization. Our two versions (LS and SVD) were tested under diverse signaling conditions and always compared very favorably to the method of [1] but at a significantly lower computational cost. At the same time the performance of our method was independent of the knowledge of the signal subspace rank, whereas both approaches in [1], [2], are extremely sensitive to the correct knowledge of this parameter.

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