Power Techniques for Blind Adaptive Channel Estimation in CDMA Systems

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Abstract— **The problem of blind adaptive channel estimation, in code-division multiple-access (CDMA) systems, is considered. Using only the spreading code of the user of interest and the received data, adaptive techniques are proposed that blindly identify the impulse response of the multipath channel. In particular, we develop RLS and LMS implementations that exhibit rapid convergence combined with low computational complexity. Both versions were inspired by the iterative power method used in numerical analysis to compute the singular vector corresponding to the largest singular value of a matrix. This is the reason why our schemes exhibit performance comparable to SVD off-line techniques while outperforming, significantly, existing adaptive methods proposed in the literature.**

I. Introduction

Code-division multiple-access (CDMA) constitutes an important emerging technology in wireless communications. In CDMA all users transmit simultaneously in time, occupy the same frequency band, but use distinct signature waveforms to allow signal separation at the receiver. This important advantage is at the same time the main source of performance degradation since to each user all other users play the role of interference.

Whenever CDMA signals propagate through a multipath environment, the effective signatures are no longer the signature waveforms but rather the convolution of these signals with the channel impulse response. This combined waveform is known as *composite signature*. It is therefore clear that if we like to apply the detection structures of [6], introduced for the non-dispersive channel case, we need to efficiently estimate the channel impulse response. Blind estimation methods tend to be, nowadays, the most frequent candidates for such a task, because of their selfsufficiency as far as training is concerned.

The blind channel estimation literature for CDMA is rather limited. In [7], [3] the (practically off-line) methods that are proposed involve a large SVD for estimating the noise subspace of the received data. This part is computationally intense, not to mention the fact that SVD presents no particular repetitive structure suitable for on-line processing. In [1] we proposed an alternative *off-line* scheme that replaces the SVD with a simple matrix power. This resulted in a significant computational gain as compared to the previous two methods, without any significant performance loss.

Blind adaptive channel estimation techniques can be found in [8], [9]. Specifically in [8], based on the analytic results offered in [5], several RLS and LMS type versions were proposed, that will be used for comparisons in our simulations section. A variant of the work in [8], reported in [9], consists in using higher order cumulants. This approach however suffers from slow convergence even for small codes, while its success relies on the Gaussian noise assumption and in particular the fact that higher order cumulants of Gaussian random variables are zero.

Here our goal is to extend the idea first proposed in [1] and use it to develop RLS and LMS adaptive algorithms. In particular we are going to introduce two versions of the power method suitably tuned for the channel estimation problem in CDMA. We recall that the power method is an iterative technique, used in numerical analysis, for computing the singular vector corresponding to the *largest* singular value of a matrix [2]. With this theory at hand we will then develop RLS and LMS type adaptive algorithms that are characterized by high performance even under very difficult signaling conditions. Compared to the corresponding versions of [8] our schemes (especially the LMS version) can perform orders of magnitude better at a similar computational cost level.

The rest of the paper is organized as follows. In Section II we introduce the signal model for synchronous CDMA, while in Section III we present two subspace problems that constitute the heart of the blind channel estimation problem. Section IV contains the power method and in particular two variants that are suitable for the solution of the two subspace problems introduced in Section III. In Section V we develop blind adaptive RLS and LMS algorithms for the channel estimation problem which we simulate in Section VI and compare to existing techniques. Finally, Section VII concludes our article.

II. Signal Model

We are focusing on the downlink scenario where all users are synchronized. Consider a K-user CDMA system with identical chip waveforms and signaling antipodally through a multipath channel in the presence of additive white (*but not necessarily Gaussian*) noise (AWN). We adopt the discrete time representation of a CDMA system that results after sampling the output of a chip matched filter applied on the received analog signal [6]. Let N be the processing gain of the code and L the length of the channel impulse response. Without loss of generality, throughout this article, we will assume that the user of interest is User-1; we will also assume that the initial delay is known and therefore we have exact synchronization with the users. The last assumption is not restrictive since, as it is indicated in [3], [1], there exist simple and reliable means for synchronization recovery.

Let $\mathbf{s}_i = [s_i(0)\, s_i(1) \cdots s_i(N-1)]^t$ be the length N normalized signature waveform of User-i (i.e. $\|\mathbf{s}_i\| = 1$), and denote by $s_i(n)$ the sequence corresponding to this signature

This work was partially supported by the European Commission through the TMR project: System Identification (FMRX-CT98-0206).

waveform zero-padded from both ends towards infinity. If $z(n)$ is the signal transmitted by the base station, then we can write

$$z(n) = \sum_{i=1}^{K} \sum_{k=-\infty}^{\infty} a_i s_i(n - kN) b_i(k), \quad (1)$$

where a_i is the amplitude of User-i, and $b_i(n)$ the corresponding bit sequence.

When the signal $z(n)$ propagates through a multipath AWN channel with impulse response $\mathbf{f} = [f(0) \ \cdots \ f(L-1)]^t$ the received signal $y(n)$ takes the form

$$\begin{aligned} y(n) &= z(n) \star f(n) + \sigma w(n) \\ &= \sum_{i=1}^{K} \sum_{k=-\infty}^{\infty} a_i \tilde{s}_i(n - kN) b_i(k) + \sigma w(n), \end{aligned} \quad (2)$$

where $\star$ stands for convolution; $\tilde{s}_i(n) = s_i(n) \star f(n)$, and σ^2 denotes the power of the additive noise.

For the presentation of our method it is more convenient to express the received signal in blocks of data. In particular we are interested in blocks of size $mN + L - 1$. For the sake of clarity however, we limit ourselves to the case $m = 1$; consequently let us consider the following block

$$\mathbf{r}_n = [y(nN) \ \cdots y((n-1)N - L + 2)]^t \quad (3)$$

which is assumed synchronized with the user of interest. Notice that due to synchronization, inside the block $\mathbf{r}_n$ exists one entire copy of the composite signature of the user of interest. To illustrate this fact and also specify in more detail the different components that make up the received signal vector, let us write $\mathbf{r}_n$ as follows

$$\mathbf{r}_n = a_1 b_1(n) \tilde{\mathbf{s}}_1 + \left(\sum_{i=2}^{K} a_i b_i(n) \tilde{\mathbf{s}}_i \right) + \text{ISI} + \sigma \mathbf{w}_n, \quad (4)$$

where $\tilde{\mathbf{s}}_i = \mathbf{s}_i \star \mathbf{f}$ is the composite signature of User-i. In (4) the first term is the signal intended for the user of interest; the next part contains terms similar to the first, but corresponding to multiaccess interference; then follows the inter-symbol interference (ISI) part, due to multipath, that includes the ISI of all users; finally the last term is the noise vector. All terms in (4) (including the ones comprising the ISI), except the additive noise, are of the form $\mathbf{d}_k b_i(n-j)$ where $\mathbf{d}_k$ suitable deterministic vectors of length $N+L-1$ and $b_i(n)$ binary data. The binary data participating in (4) are mutually independent and also independent from the noise vector $\mathbf{w}_n$.

One final point we should make, before proceeding with the presentation of the two subspace problems, is the fact that the composite signature of User-1 can be written as

$$\tilde{\mathbf{s}}_1 = \mathbf{S}_1 \mathbf{f} \quad (5)$$

where $\mathbf{S}_1$ is a convolution matrix of size $(N + L - 1) \times L$, corresponding to the initial signature of User-1 and defined as

$$\mathbf{S}_1 = \begin{bmatrix} s_1(0) & 0 & \cdots & 0 \\ \vdots & s_1(0) & \ddots & \vdots \\ s_1(N-1) & \vdots & \ddots & 0 \\ 0 & s_1(N-1) & \ddots & s_1(0) \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & s_1(N-1) \end{bmatrix}. \quad (6)$$

III. Two Subspace Problems

Let us now see how we can identify the channel impulse response $\mathbf{f}$ when the data autocorrelation matrix is available. Therefore let

$$\mathbf{R} \triangleq \mathbb{E}\{\mathbf{r}_n \mathbf{r}_n^t\} = \mathbf{Q} + \sigma^2 \mathbf{I} \quad (7)$$

where $\mathbf{Q} = \sum \mathbf{d}_k \mathbf{d}_k^t$ is a symmetric, nonnegative definite matrix, of dimensions $N + L - 1$, formed by the $\mathbf{d}_k$ vectors introduced in the signal model.

By performing an SVD on the autocorrelation matrix $\mathbf{R}$ we can write

$$\mathbf{R} = [\mathbf{U}_s \quad \mathbf{U}_w] \begin{bmatrix} \mathbf{\Lambda}_s + \sigma^2 \mathbf{I} & \mathbf{0} \\ \mathbf{0} & \sigma^2 \mathbf{I} \end{bmatrix} [\mathbf{U}_s \quad \mathbf{U}_w]^t, \quad (8)$$

where $\mathbf{U}_s, \mathbf{U}_w$ are orthonormal bases for the signal and noise subspace respectively. In particular we should note that $\mathbf{U}_w$ spans the subspace corresponding to the *smallest* singular value of $\mathbf{R}$ (which is equal to σ^2). Due to the orthogonality of the two subspaces, for any vector $\mathbf{d}_k$ in the signal subspace we have

$$\mathbf{U}_w^t \mathbf{d}_k = 0. \quad (9)$$

Since the composite signature $\tilde{\mathbf{s}}_1$ of the user of interest constitutes one such vector, from (5) and (9) we deduce that

$$\mathbf{U}_w^t \tilde{\mathbf{s}}_1 = \mathbf{U}_w^t \mathbf{S}_1 \mathbf{f} = \left(\mathbf{S}_1^t \mathbf{U}_w \mathbf{U}_w^t \mathbf{S}_1 \right) \mathbf{f} = 0. \quad (10)$$

Equ. (10) suggests the recovery of $\mathbf{f}$ as the singular vector corresponding again to the *smallest* singular value (which is equal to zero) of the matrix $\mathbf{S}_1^t \mathbf{U}_w \mathbf{U}_w^t \mathbf{S}_1$. Equ. (10) constitutes *the heart* of all existing SVD based methods and enjoys the same property here.

As it becomes clear from the preceding discussion, there are two subspace problems involved in (10). The first concerns the estimation of $\mathbf{U}_w$ and the second the recovery of the channel impulse response $\mathbf{f}$. Let us present the two problems more explicitly.

Problem 1: If $\mathbf{R}$ satisfies the decomposition in (8) we are interested in estimating the *product* $\mathbf{U}_w \mathbf{U}_w^t$ where $\mathbf{U}_w$ is an orthonormal basis for the noise subspace corresponding to the *smallest* singular value σ^2 of $\mathbf{R}$.

Problem 2: If $\mathbf{R}$ and $\mathbf{U}_w$ are as in Problem 1 and $\mathbf{S}_1$ the matrix defined in (6), we are interested in estimating the singular vector $\mathbf{f}$ corresponding to the *smallest* singular value of the matrix

$$\mathbf{W} = \mathbf{S}_1^t \mathbf{U}_w \mathbf{U}_w^t \mathbf{S}_1. \quad (11)$$

In [7] both problems are solved by direct SVD while in [3] the first with SVD and the second with QR. It is clear that applying SVD on $\mathbf{R}$ (or more accurately to its estimate) to recover $\mathbf{U}_w$ is computationally heavy and disqualifies these methods from on-line processing. We should also mention the need of these approaches in *knowing the noise subspace rank*. It turns out [1] that even slight errors in the estimate of this parameter, can produce significant performance degradation in the schemes proposed in [7], [3]. This becomes particularly serious since existing rank estimation techniques are not characterized by extreme robustness. In [1] a power method was proposed to replace the large SVD of [7], [3]. This idea will be fully exploited in the next section in a direction that is suitable for both subspace problems introduced previously.

IV. Power Method Variants

The power method [2] is a simple *iterative* technique that can be used to provide estimates of the subspace corresponding to the *largest* singular value of a matrix. Let us present two variants of this method that are appropriate for solving the two subspace problems of interest and that will also serve as a base for developing our adaptive algorithms.

Lemma 1: Let $\mathbf{R}$ be as in (7) and (8) and $\rho \geq 0$ a nonnegative scalar, we then have

$$\lim_{k\to\infty}\left(\frac{\rho\mathbf{I}+\mathbf{R}}{\rho+\sigma^2}\right)^{-k} = \mathbf{U}_w\mathbf{U}_w^t. \tag{12}$$

Proof: The proof is straightforward, it basically uses the decomposition of $\mathbf{R}$ defined in (8). ■

It is clear that Lemma 1 contributes to the solution of the first subspace problem, i.e. the estimation of the product $\mathbf{U}_w\mathbf{U}_w^t$ required in (10).

Lemma 2: Let $\mathbf{W}$ be the matrix defined in (11); suppose that the vector $\mathbf{f}$ which satisfies (10) is unique and of unit norm, then with $\alpha = 1/\mathrm{tr}\{\mathbf{W}\}$ the sequence of vectors $\mathbf{f}_n$ defined as

$$\mathbf{f}_n = (\mathbf{I}-\alpha\mathbf{W})\mathbf{f}_{n-1}/\|(\mathbf{I}-\alpha\mathbf{W})\mathbf{f}_{n-1}\| \tag{13}$$

converges to the channel impulse response $\pm\mathbf{f}$ (modulo a sign ambiguity), provided that $\mathbf{f}_0$ is not orthogonal to $\mathbf{f}$.

Proof: Again the proof presents no particular difficulty. One can show that $\mathbf{f}_n = (\mathbf{I}-\alpha\mathbf{W})^n\mathbf{f}_0/\|(\mathbf{I}-\alpha\mathbf{W})^n\mathbf{f}_0\|$. Since $\mathbf{W}\mathbf{f} = 0$ this means that $\mathbf{f}$ is a singular vector for the matrix $\mathbf{I}-\alpha\mathbf{W}$ corresponding to the unit singular value (which is the largest since $\alpha\mathbf{W}$ is nonnegative definite with all singular values smaller than unity). Using singular value decomposition we can then see that $\lim_{n\to\infty}(\mathbf{I}-\alpha\mathbf{W})^n = \mathbf{f}\mathbf{f}^t$, which yields

$$\lim_{n\to\infty}\mathbf{f}_n = \mathrm{sgn}(\mathbf{f}^t\mathbf{f}_0)\mathbf{f}. \tag{14}$$

This concludes the proof. ■

Lemma 2 contributes to the solution of the second subspace problem and will clearly provide channel impulse response estimates. At this point we can make the following important remarks.

Remark 1: In Lemma 1 the convergence in (12) is exponential and we can show that the corresponding rate is maximized when $\rho = 0$. Regardless of this fact, the employment of a $\rho > 0$ in the scheme turns out to be particularly useful in the case of the LMS version since it allows the algorithm to *forget past data* exactly as in the exponentially windowed RLS case. This desirable property is *not* enjoyed by our LMS scheme when $\rho = 0$. In the exponentially windowed RLS version, on the other hand, we can select $\rho = 0$ since this form of RLS has a a natural ability to forget past data.

Remark 2: A subtle and very important remark regarding Lemma 1 concerns the employment of power k. Notice that the limit is correct, i.e. we obtain the product $\mathbf{U}_w\mathbf{U}_w^t$, *only* when the singular values corresponding to the noise subspace are *exactly* equal. Unfortunately in a realistic situation, when only estimates of $\mathbf{R}$ are available, this is rarely the case. This has a grave consequence since the corresponding limit instead of being the desired product will become just the rank-one matrix $\mathbf{u}\mathbf{u}^t$ where $\mathbf{u}$ is the singular vector corresponding to the smallest singular value of the *estimate* of $\mathbf{R}$. This in turn will directly lead to wrong estimates for the impulse response $\mathbf{f}$ since $\mathbf{W}$ will be of rank one as well and thus $\mathbf{f}$ will no longer be the only vector satisfying (10).

Fortunately, for CDMA signals, there is a simple solution to this problem. In [1] it was observed that, for off-line processing, it was sufficient to use powers up to $k = 3$ and practically match the performance of the direct SVD based techniques. We are going to follow the same idea here. In other words we are going to approximate the product $\mathbf{U}_w\mathbf{U}_w^t$ as follows

$$\widehat{\mathbf{U}_w\mathbf{U}_w^t} = \left(\frac{\rho\mathbf{I}+\mathbf{R}}{\rho+\sigma^2}\right)^{-k}, \quad k = 1,2,3. \tag{15}$$

Remark 3: Our final remark concerns the usage of (15). Notice that in approximating the product $\mathbf{U}_w\mathbf{U}_w^t$ this way, *we do not need any knowledge of the noise subspace rank*. This is particularly desirable since, as we previously explained, rank estimate methods tend to be complicated and non-robust.

We are now ready to proceed with the presentation of our blind adaptive schemes.

V. Blind Adaptations for Channel Estimation

As stated in Problem 2, the channel impulse response can be recovered as the singular vector corresponding to the smallest singular value of the matrix $\mathbf{W} = \mathbf{S}_1^t\mathbf{U}_w\mathbf{U}_w^t\mathbf{S}_1$. Using the approximation proposed in (15) we have the following estimate for this matrix

$$\hat{\mathbf{W}}(k) = \mathbf{S}_1^t(\rho\mathbf{I}+\mathbf{R})^{-k}\mathbf{S}_1, \tag{16}$$

where we have eliminated the quantity $(\rho+\sigma^2)^k$ because scalars do not play any role in the subspace determination problem.

Since the autocorrelation matrix $\mathbf{R}$ is not available, we are interested in producing, adaptively, estimates $\hat{\mathbf{W}}_n(k)$ of the matrix $\hat{\mathbf{W}}(k)$ defined in (16). There are different possibilities that we exploit next. Notice however that with the help of any such estimate $\hat{\mathbf{W}}_n(k)$ the power method presented in Lemma 2 can be replaced by

$$\hat{\mathbf{f}}_n = (\mathbf{I}-\alpha_n\hat{\mathbf{W}}_n(k))\hat{\mathbf{f}}_{n-1}/\|(\mathbf{I}-\alpha_n\hat{\mathbf{W}}_n(k))\hat{\mathbf{f}}_{n-1}\| \tag{17}$$

where $\alpha_n = 1/\mathrm{tr}\{\hat{\mathbf{W}}_n(k)\}$. In other words, at every step, we first *adapt* $\hat{\mathbf{W}}_n(k)$ and then apply a *single iteration* of the power method. Let us now see what possibilities exist for the estimate $\hat{\mathbf{W}}_n(k)$.

A. Channel Estimation via RLS

As was mentioned previously here we select $\rho = 0$ and for the adaptive estimate $\hat{\mathbf{W}}_n(k)$ of the matrix $\hat{\mathbf{W}}(k)$ we propose

$$\hat{\mathbf{W}}_n(k) = \mathbf{S}_1^t(\hat{\mathbf{R}}_n^{-1})^k\mathbf{S}_1, \quad k = 1, 2, 3. \tag{18}$$

where $\hat{\mathbf{R}}_n$ is the exponentially windowed sample autocorrelation matrix of the data $\mathbf{r}_n$, i.e. $\hat{\mathbf{R}}_n = \sum_{i=0}^{n} \lambda^{n-i}\mathbf{r}_i\mathbf{r}_i^t$, with $0 < \lambda < 1$, a forgetting factor. The overall complexity for computing $\hat{\mathbf{W}}_n(k)$ and applying one step of (17) can be shown to be $(5+2kL)(N+L-1)^2+(2L^2+3)(N+L-1)+O(L^2)$.

We should mention that our RLS version is similar to the one in [10] and when $k = 1$ to the RLS proposed in [8]. The advantage here is that we avoid even the small SVD on $\hat{\mathbf{W}}_n(k)$ used in [8], [10], since we replace it with *one step* of the simple power recursion in (17). Furthermore, as far as the method of [8] is concerned, as we are going to see in the simulations part, by employing higher values of the power k we can ameliorate performance significantly.

B. Channel Estimation via Leakage LMS

This is the most practically important part of our work. The LMS scheme we are going to present is computationally simple with performance that can be orders of magnitude better than the corresponding LMS adaptation of [8].

An alternative means to generate estimates for $\hat{\mathbf{W}}(k)$ consists in writing

$$\hat{\mathbf{W}}(k) = \mathbf{S}_1^t\hat{\mathbf{V}}(k) \tag{19}$$

where

$$\hat{\mathbf{V}}(k) = (\rho\mathbf{I} + \mathbf{R})^{-k}\mathbf{S}_1 \tag{20}$$

and produce estimates $\hat{\mathbf{V}}_n(k)$ for $\hat{\mathbf{V}}(k)$. It turns out that LMS is particularly suited for this task. Consider the following system of adaptations for $i = 1, 2, \ldots, k$

$$\hat{\mathbf{V}}_n(i) = \lambda\hat{\mathbf{V}}_{n-1}(i) + \mu\left(\hat{\mathbf{V}}_{n-1}(i-1) - \mathbf{r}_n\mathbf{r}_n^t\hat{\mathbf{V}}_{n-1}(i)\right) \tag{21}$$

where $\hat{\mathbf{V}}_n(0) = \hat{\mathbf{V}}_0(i) = \mathbf{S}_1$, and $0 < \lambda < 1$, a forgetting factor. One can then show, using standard independence assumption arguments, that $\hat{\mathbf{V}}_n(k)$ converges in the mean to

$$\lim_{n\to\infty} \mathbb{E}[\hat{\mathbf{V}}_n(k)] = \left(\frac{1-\lambda}{\mu}\mathbf{I} + \mathbf{R}\right)^{-k}\mathbf{S}_1. \tag{22}$$

which coincides with (20) with $\rho = (1-\lambda)/\mu$. We thus conclude that $\hat{\mathbf{V}}_n(k)$, computed with (21), can provide estimates for $\hat{\mathbf{V}}(k)$.

Using $\hat{\mathbf{V}}_n(k)$ we can now obtain estimates for $\hat{\mathbf{W}}(k)$ following (19) as $\hat{\mathbf{W}}_n(k) = \mathbf{S}_1^t\hat{\mathbf{V}}_n(k)$; finally we apply one iteration of the power method in (17) to obtain the estimate $\hat{\mathbf{f}}_n$ of the channel impulse response. The overall computational complexity, with careful housekeeping, becomes $9kL(N+L-1)+O(kL^2)$, which is an order of magnitude smaller than the RLS version and two orders smaller than the direct SVD based techniques of [7], [3].

Due to the application of the forgetting factor λ, as was mentioned before, the LMS algorithm is capable of forgetting past information, exponentially. The corresponding algorithm is known in the literature as *Leakage LMS*.

VI. Simulations - Comparisons

In this section, we provide several simulation results to demonstrate the performance of the blind adaptive schemes developed previously. In particular we compare our RLS and LMS implementations against the corresponding schemes proposed in [8].

Randomly generated sequences of length $N = 128$ are used as spreading codes. Once generated, the codes are kept constant for the whole simulation set. Moreover, all graphs presented in the figures are the result of an average of 100 independent runs. In each run we apply three different abrupt changes in order to observe the ability of the corresponding algorithms to follow them. Specifically at bit 5000 we change the channel, and at bits 10000 and 15000 the number of users. For the multipath channel we start with the length 3 "difficult" channel and at 5000 we switch to the length 10 "easy" channel of [4]. For our estimation on the other hand we assume that we have available only an upper bound for the channel length which is $L = 10$. In other words even the length 3 channel is identified as being of length 10.

The signaling conditions are the following: we start with $K = 55$ users, under perfect power control. At bit 10000 ten additional users enter the channel, 5 of them having power equal to the user of interest and the remaining 5 being 10 db stronger. Finally, at bit 15000 the last 10 users along with 5 more exit the channel.

Fig. 1 depicts the mean square channel estimation error of the RLS schemes when the SNR of the user of interest is equal to 20 db. We can see that our $k = 1$ version practically matches the RLS of [8] without needing an SVD on the matrix $\mathbf{W}_n(1)$ at each step. By employing higher powers $k = 2, 3$ there is a slight performance improvement only at the beginning. After the channel changes at bit 5000 all RLS algorithms converge quickly to their new steady state. It is clear that in this high SNR environment selecting $k = 1$ is sufficient.

Fig. 2 presents the performance of the corresponding LMS versions for exactly the same signaling conditions as in the previous example. We clearly see that, in contrast to the RLS case, here there are substantial performance gains by employing higher powers. In particular the LMS method of [8] has an overall performance which is 20 db inferior to our $k = 3$ version.

Next we consider the same signaling scenario but with a significantly lower SNR. Specifically we set the desired user's SNR at 10 db. The performance of the RLS schemes is presented in Fig. 3 while the corresponding of the LMS in Fig. 4. Again the method of [8] is identical to our RLS $k = 1$ version. Here however, in this low SNR environment, employing higher orders of k ameliorates the overall RLS performance significantly. This is particularly apparent in the initial part, in the case of the "difficult" channel. In Fig. 4, our LMS version for $k = 3$, outperforms again the LMS algorithm of [8] by 10 db.

A point that should be mentioned concerns the initial performance (up to bit 5000) of all channel estimation schemes. In fact, if we had exact knowledge of the filter length, that is, if we had used $L = 3$ instead of $L = 10$, both LMS and RLS methods would have attained better performance levels than the ones depicted in the corresponding figures. Since this

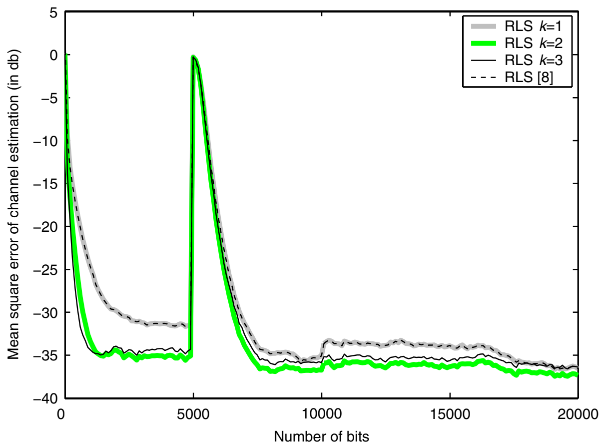

Fig. 1. Performance of the proposed RLS channel estimation schemes versus the method of [8]; noise power 20 db.

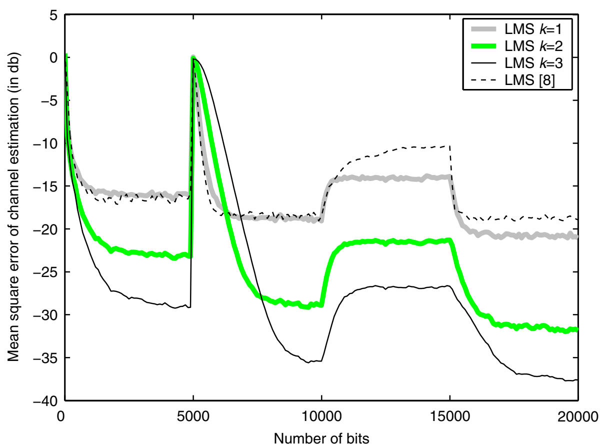

Fig. 2. Performance of the proposed LMS channel estimation schemes versus the method of [8]; noise power 20 db.

information is not a-priori available, it is preferable to use a common L which is an upper bound for all cases. Finally, comparing the RLS with the LMS schemes we clearly observe the considerably more robust behavior of the former which, unfortunately, comes at a much greater computational cost.

VII. Conclusion

In this paper, we have considered the problem of blind adaptive channel estimation in CDMA systems. RLS and LMS type algorithms were developed based on the iterative power method which is known, from numerical analysis, to yield efficient subspace estimates. With a number of simulations we demonstrate the satisfactory performance of the proposed algorithmic versions in a dynamic environment with variations in the multipath channel and the number of users. Our algorithms have been compared against the corresponding schemes of [8] and asserted to offer significant performance gains under various signaling scenarios.

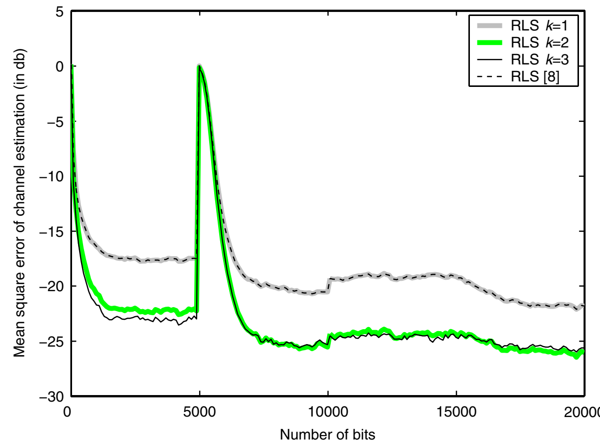

Fig. 3. Performance of the proposed RLS channel estimation schemes versus the method of [8]; noise power 10 db.

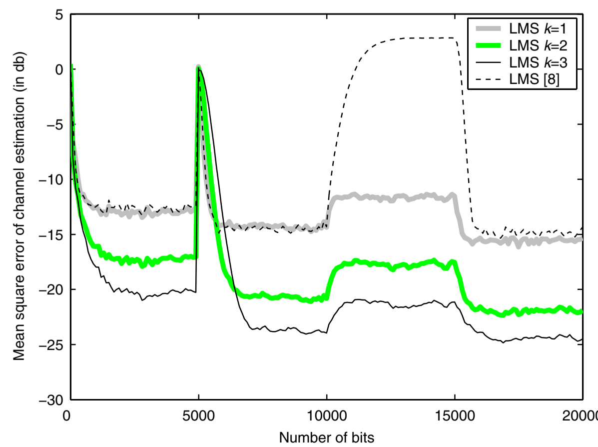

Fig. 4. Performance of the proposed LMS channel estimation schemes versus the method of [8]; noise power 10 db.

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