Adaptive Algorithms for Blind Channel Estimation in OFDM Systems

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Abstract— **The problem of blind adaptive channel estimation in OFDM systems is considered. Focusing on the zero padding approach, for the first time adaptive algorithms are proposed that blindly identify the impulse response of the multipath channel. In particular, we develop RLS and LMS schemes that exhibit rapid convergence combined with low computational complexity. Both versions are obtained by properly modifying the orthogonal iteration, a method used in Numerical Analysis for the computation of singular vectors. With a number of simulations we demonstrate the satisfactory performance of our adaptive schemes under diverse signaling conditions.**

I. Introduction

ORTHOGONAL Frequency Division Multiplexing (OFDM) constitutes a promising technology for high speed transmission in frequency selective fading environment [1]. OFDM presents several important advantages, some of which are: high spectral efficiency; simple implementation (with IDFT/DFT pairs); mitigation of intersymbol interference (ISI) and robustness to frequency selective fading environments. Inevitably, these desirable characteristics contribute towards a continuously rising interest for OFDM. We should also mention that OFDM has been selected for the European standard of digital audio and video broadcasting, digital subscriber lines and wireless local area networks.

In practice OFDM systems operate over a dispersive channel and therefore a guard interval, no smaller than the anticipated channel spread, is usually inserted in the transmitted sequence. As far as this guard period is concerned, there exist two alternatives. The first, known as cyclic prefix (CP), consists in re-transmitting inside the guard interval the initial portion of the transmitted sequence, while the second, known as zero padding (ZP), transmits no information during the same interval. In this work, we mainly focus on the latter.

The ZP approach is very appealing [10] and has started gaining popularity mainly due to its simplicity. Its strongest point is the *complete elimination* of inter-block interference (IBI), which allows for a number of interesting detection structures. In [10] a detailed comparison between CP-OFDM and ZP-OFDM receivers is offered and several merits of the ZP approach are emphasized.

In coherent detection and adaptive loading, knowledge of the channel impulse response is imperative. Since the channel state information is usually unknown to the receiver, it needs to be efficiently estimated. Channel estimation techniques can be roughly divided into two major categories the *supervised* or *trained* and the *unsupervised* or *blind*. The first requires training/pilot sequences whereas the latter uses only the received data. Due of course to their self-sufficiency in training, blind techniques are considered more attractive than their trained counterparts; they tend however to be heavier from a computational complexity point of view.

An additional property that currently distinguishes the two categories is the existence of adaptive schemes for the implementation of the corresponding channel estimation methods. Whenever adaptivity is involved this usually results in a significant computational gain as compared to off-line techniques; moreover the computation is repetitive and uniformly spread over time. The latter characteristic is very appealing for DSP implementations. We should, of course, not forget the ability of adaptive techniques to follow changes in the characteristics of the received signal, which here translates into following changes in the channel impulse response. Although one can find adaptive schemes for trained channel identification methods, this is not the case for blind approaches. Existing blind OFDM channel identification methods are mainly off-line.

The majority of articles concerning the problem of channel estimation in OFDM systems, uses pilot tones or training sequences [2], [4]. In [8] a comparative study of non-blind methods can be found. Even though the pilot-aided literature is rich, we will not pursue its presentation any further, since our main interest lies with blind channel identification methods. Regarding blind techniques, in [6] blind channel identification is performed by exploiting the cyclostationarity present in CP-OFDM. In [9] a subspace approach is proposed for channel estimation that takes advantage of the redundancy existing in CP-OFDM. An alternative subspace approach is presented in [7] which extends the previous idea by incorporating virtual carriers inside the OFDM transmitted block. The two latter methods require singular value decomposition (SVD) of the received data autocorrelation matrix and are therefore characterized by high computational cost. We should stress that SVD is known to lack a repetitive structure that could lead to efficient adaptive implementations.

In this work we exploit the subspace method in order to develop *adaptive* algorithms for blind channel identification in ZP-OFDM systems. To our knowledge, this is the first time such schemes are proposed for OFDM systems. Specifically we are going to develop RLS and LMS type algorithms that can solve very efficiently the channel estimation problem. Both versions have significantly lower computational complexity as compared to the direct SVD approaches of [7], [9]. In particular our LMS version is extremely simple with a computational complexity that is almost two orders of magnitude smaller than a direct SVD approach. We should finally mention that

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the development of our adaptive schemes was possible by properly modifying the *orthogonal iteration* used in Numerical Analysis for computing the subspace corresponding to the largest singular values of a matrix [5].

The rest of the paper is organized as follows. Section II introduces the signal model for a ZP-OFDM system. We continue in Section III with the definition of two subspace problems that constitute the heart of the blind channel estimation methodology. The same section contains also the orthogonal iteration suitably tuned for the solution of the two subspace problems in question. In Section IV we develop blind adaptive RLS and LMS algorithms for the identification of the channel impulse response. Simulation results are offered in Section V, and finally Section VI concludes our article.

II. System Model

OFDM modulation has the characteristic of multiplexing data symbols over a large number of orthogonal carriers. Consider an OFDM system where the guard interval consists of a zero padded sequence. Fig. 1 depicts the baseband discrete-time block equivalent model of a standard ZP-OFDM transmitter. Let each information block be comprised of N

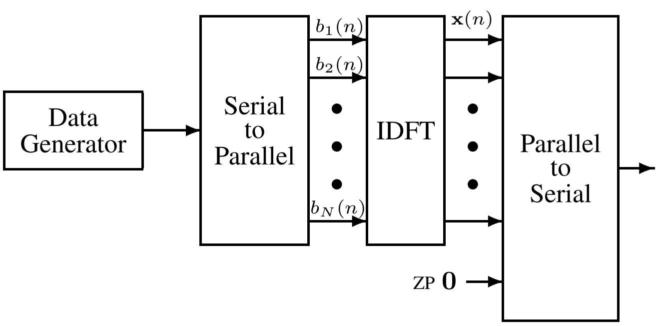

Fig. 1. Discrete time block ZP-OFDM transmitter.

symbols and denote by L the size of the ZP. The n-th length-N block $\mathbf{b}_N(n) = [b_1(n) \ \dots \ b_N(n)]^T$ passes through a serial to parallel converter and then it is modulated by IDFT. After the IDFT, a sequence of L zeros (zero padding) is inserted between two consecutive blocks to form the transmitted vector $\mathbf{x}(n)$. The latter is of length $N+L$, and can be put under the following form

$$\mathbf{x}(n) = \begin{bmatrix} \mathbf{F}_N^H \\ \mathbf{0}_{L\times N} \end{bmatrix} \mathbf{b}_N(n), \tag{1}$$

where $\mathbf{F}_N$ stands for the DFT matrix defined as

$$\mathbf{F}_N = \frac{1}{\sqrt{N}} \begin{bmatrix} 1 & 1 & 1 & \cdots & 1 \\ 1 & W_N & W_N^2 & \cdots & W_N^{N-1} \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ 1 & W_N^{N-1} & W_N^{2(N-1)} & \cdots & W_N^{(N-1)(N-1)} \end{bmatrix}, \tag{2}$$

with $W_N = e^{-j\frac{2\pi}{N}}$; superscript "^{H}" denotes conjugate-transpose and $\mathbf{0}_{L\times N}$ is a zero matrix of dimensions $L \times N$. The parallel block $\mathbf{x}(n)$ is finally transformed into a serial sequence in order to be transmitted through the channel.

The transmitted signal propagates through a multipath additive white noise (AWN), *not necessarily Gaussian*, channel with impulse response $\mathbf{h} = [h_1 \ \dots \ h_{L+1}]^T$. Here we have assumed that the channel has a finite impulse response of length at most $L+1$. Such an assumption is very common in OFDM systems and constitutes the main reason for introducing the guard interval in the great majority of OFDM models. Whenever ZP is employed, after assuming synchronization with the transmitted sequence, the n-th received data block $\mathbf{y}(n)$ of length $N+L$ can be expressed as

$$\mathbf{y}(n) = \mathbf{H}\mathbf{F}_N^H \mathbf{b}_N(n) + \mathbf{w}(n). \tag{3}$$

In the above relation $\mathbf{w}(n)$ is an AWN vector of length $N+L$ with i.i.d. zero-mean elements of variance equal to σ^2, which are also independent of the transmitted symbols $\mathbf{b}_N(n)$; finally, matrix $\mathbf{H}$ is a convolution matrix of dimensions $(N+L)\times N$ defined as follows.

$$\mathbf{H} = \begin{bmatrix} h_1 & 0 & \cdots & 0 \\ \vdots & h_1 & \ddots & \vdots \\ h_{L+1} & \vdots & \ddots & 0 \\ 0 & h_{L+1} & \ddots & h_1 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & h_{L+1} \end{bmatrix} \tag{4}$$

We can now verify the very interesting property of the ZP-OFDM model stated in the Introduction, that is, its ability to completely eliminate the IBI between consecutive blocks. This fact becomes evident from the expression depicted in (3) where there are no other data involved except the current information block $\mathbf{b}_N(n)$. In order to obtain the same property in CP-OFDM, in every received data block of size $N+L$ we need to *discard* the first L data samples and use the remaining N, thus throwing away information that could be useful. Consequently, in ZP-OFDM, the entire linear convolution of each transmitted block with the channel impulse response is preserved without being altered by IBI. This important property is in fact behind the successful application of the subspace method proposed in the next section.

III. Main Idea

In this section we are focusing on how to identify the channel impulse response when the received data autocorrelation matrix is available. As it is almost always the case with subspace techniques, the key idea consists in properly decomposing the data into the signal and noise subspace and then defining suitable subspace determination problems that lead to the final estimate of the channel impulse response.

A. A Subspace Approach

Consider the autocorrelation matrix $\mathbf{R}$ of the received data vector $\mathbf{y}(n)$ defined in (3). Assuming that the elements of $\mathbf{b}_N(n)$ are i.i.d. and using the fact that the DFT matrix $\mathbf{F}_N$ defined in (2) is orthonormal, we conclude that

$$\mathbf{R} \triangleq \mathbb{E}\{\mathbf{y}(n)\mathbf{y}^H(n)\} = \mathbf{H}\mathbf{H}^H + \sigma^2\mathbf{I}_{N+L}, \tag{5}$$

where σ^2 the AWN power and $\mathbf{I}_K$ denotes the identity matrix of size K. The matrix $\mathbf{H}\mathbf{H}^H$ is Hermitian and nonnegative definite, of dimensions $(N+L)\times(N+L)$. From (3) we also conclude that the columns of $\mathbf{H}$ span the signal subspace. If the channel impulse response $\mathbf{h}$ is not identically zero then, since $\mathbf{H}$ is a convolution matrix of the form of (4), it is also of full column rank. This suggests that the signal subspace has rank equal to N while its complement, the noise subspace, has rank equal to L.

By performing an SVD on the autocorrelation matrix $\mathbf{R}$ and taking into account the previous observation we can write

$$\mathbf{R} = [\mathbf{U}_s \quad \mathbf{U}_n] \begin{bmatrix} \mathbf{\Lambda}_s + \sigma^2 \mathbf{I}_N & \mathbf{0} \\ \mathbf{0} & \sigma^2 \mathbf{I}_L \end{bmatrix} [\mathbf{U}_s \quad \mathbf{U}_n]^H, \qquad (6)$$

where $\mathbf{U}_s, \mathbf{U}_n$ are orthonormal bases for the signal and noise subspace respectively and $\mathbf{\Lambda}_s$ is a diagonal matrix with positive elements of size N. We should note that $\mathbf{U}_s$ is in fact an orthonormalized version of the columns of the matrix $\mathbf{H}$; $\mathbf{U}_n$ on the other hand is a basis for the subspace which is orthogonal to the columns of $\mathbf{U}_s$ or equivalently to the columns of $\mathbf{H}$. Finally notice that $\mathbf{U}_n$ involves the singular vectors of the matrix $\mathbf{R}$ corresponding to its *smallest* singular value (which is equal to σ^2).

Let $\mathbf{v} = [v_1 \ \cdots \ v_{N+L}]^T$ be a vector of length $N+L$ that belongs to the noise subspace. Due to the orthogonality of the two subspaces we can then write

$$\mathbf{v}^H \mathbf{U}_s = \mathbf{v}^H \mathbf{H} = 0. \qquad (7)$$

Since $\mathbf{H}$ has the Toeplitz form of (4), the vector-matrix product $\mathbf{v}^H\mathbf{H}$ can be written alternatively as

$$\mathbf{v}^H \mathbf{H} = \mathbf{h}^T \mathbf{V}^\star = 0, \qquad (8)$$

where superscript "$\star$" denotes complex conjugate and $\mathbf{V}$ is the following Hankel matrix of dimensions $(L+1)\times N$ made up from the elements of the vector $\mathbf{v}$

$$\mathbf{V} = \begin{bmatrix} v_1 & v_2 & \cdots & v_N \\ v_2 & v_3 & \cdots & v_{N+1} \\ \vdots & \vdots & & \vdots \\ v_{L+1} & v_{L+2} & \cdots & v_{N+L} \end{bmatrix}. \qquad (9)$$

By taking the complex conjugate of the relation in (8) we end up with the following equation.

$$\mathbf{h}^H \mathbf{V} = 0 = \mathbf{h}^H \mathbf{V}\mathbf{V}^H \mathbf{h} \qquad (10)$$

Let $\mathbf{U}_n = [\mathbf{v}_1 \ldots \mathbf{v}_L]$ and $\mathbf{V}_i$ be the Hankel versions of $\mathbf{v}_i$, then since (10) holds for every vector $\mathbf{v}$ in the noise subspace we have

$$\mathbf{h}^H \mathbf{W} \mathbf{h} = 0, \quad \text{with} \quad \mathbf{W} = \sum_{i=1}^{L} \mathbf{V}_i \mathbf{V}_i^H. \qquad (11)$$

It is actually (11) that will be used to determine the channel impulse response $\mathbf{h}$. Indeed from (11) we can see that $\mathbf{h}$ can be recovered as the singular vector corresponding to the *smallest* singular value (which is zero) of the matrix $\mathbf{W}$. Key point of course in estimating $\mathbf{h}$ through the subspace problem defined in (11) is the formation of the matrix $\mathbf{W}$. This is possible if we have available a basis of the noise subspace $\mathbf{U}_n$ of the data autocorrelation matrix $\mathbf{R}$.

From the preceding discussion we conclude that the complete channel estimation problem involves two similar subspace determination problems, which we would like to present more explicitly next.

Subspace Problem 1: The first step in applying (11) is the determination of the noise subspace $\mathbf{U}_n$ of the data autocorrelation matrix $\mathbf{R}$. The basis $\mathbf{U}_n$ is of dimensions $L\times(N+L)$ and can be seen as a collection of L orthonormal vectors corresponding to the smallest singular value σ^2 of $\mathbf{R}$ as shown in (6).

Subspace Problem 2: Once $\mathbf{U}_n$ is estimated from the first problem, its columns define the matrices $\mathbf{V}_i$ that compose $\mathbf{W}$. The channel impulse response $\mathbf{h}$ can then be recovered as the singular vector corresponding to the *smallest* singular value of the matrix $\mathbf{W}$.

Both problems involve the determination of a subspace corresponding to the smallest singular value of a matrix. Although similar methodology has been developed for channel estimation in CDMA there exists a major difference that distinguishes the current setting from the one used in CDMA. Here we know exactly the noise subspace rank while this is not the case in CDMA where this parameter is variable, and depends on the number of users in the channel [3]. Due to this extra knowledge it will be possible to develop algorithms for OFDM that are more powerful than their CDMA counterparts.

B. The Orthogonal Iteration

The orthogonal iteration [5], [11] is a simple iterative technique that can be used to provide estimates of subspaces corresponding to the *largest* singular value(s) of a matrix. As we have seen in the two problems previously introduced, in both cases we are interested in obtaining the subspace (or singular vector) corresponding to the smallest singular value. Let us therefore present two suitable variants of the orthogonal iteration that achieve this goal.

Lemma 1: Suppose that matrix $\mathbf{\Omega}$ is Hermitian, positive definite and of dimensions $K\times K$. Denote by $\omega_1 \geq \cdots \geq \omega_J > \omega_{J+1} \geq \cdots \geq \omega_K > 0$ its singular values and by $\mathbf{f}_1, \ldots, \mathbf{f}_K$ the corresponding singular vectors. Define the sequence of matrices $\{\mathcal{S}(k)\}$, $k = 1, 2, \ldots,$ of dimensions $K\times(K-J)$ by either of the two iterations

$$\mathcal{S}(k) \;=\; \text{orthonorm}\{\mathbf{\Omega}^{-1}\mathcal{S}(k-1)\} \qquad (12)$$

$$\mathcal{S}(k) \;=\; \text{orthonorm}\{(\mathbf{I} - \bar{\mu}\mathbf{\Omega})\mathcal{S}(k-1)\} \qquad (13)$$

where "orthonorm" stands for orthonormalization using QR-decomposition; $\bar{\mu} = \mu/\text{trace}\{\mathbf{\Omega}\}$ and $0 < \mu \leq 1$.

If $\mathcal{S}(0)$ is such that $\mathcal{S}(0)^H[\mathbf{f}_{J+1}\cdots\mathbf{f}_K]$ is *invertible*, then $\mathcal{S}(k) \to [\mathbf{f}_{J+1}\cdots\mathbf{f}_K]$ as $k\to\infty$.

Proof: A proof for the convergence of the orthogonal iteration can be found in [11, pages 296-297]. ■

If certain singular values coincide then the corresponding singular vectors are not unique. In this case the orthogonal iteration converges to a basis in the corresponding subspace. Finally, more details regarding the QR factorization can be found in [5].

It is possible to prove that our method is *consistent*; i.e. for the combination of the two subspace problems, there exists a unique solution (modulo a complex scalar ambiguity), which is necessarily equal to the channel impulse response. The ambiguity manifesting itself in our methodology is also present in [7], [9]. It turns out that it can be removed by incorporating pilot symbols. In fact, even a single pilot symbol is sufficient. Unfortunately, due to lack of space, we will not detail on these two important issues.

IV. Adaptive Algorithms

Let us now see how, the results presented in the previous section, can be used to solve, adaptively, the two subspace problems and yield the necessary estimates for the channel impulse response. The two approaches we are going to present (RLS and LMS) differ in the way they provide adaptive estimates of the noise subspace $\mathbf{U}_n$ of $\mathbf{R}$. Therefore, we firstly concentrate on Subspace Problem 2, since it is solved in the same way for both implementations.

Assuming that at each time instant, estimates $\mathbf{U}_n(n)$ of the noise subspace are available, we can form the Hankel matrices $\mathbf{V}_i(n)$ $i = 1, \ldots, L$, following (9) and thus produce an estimate

$$\mathbf{W}(n) = \sum_{i=1}^{L} \mathbf{V}_i(n)\mathbf{V}_i^H(n) \qquad (14)$$

for the matrix $\mathbf{W}$ which enters in the second subspace problem. After computing $\mathbf{W}(n)$ we can then obtain an adaptive estimate $\mathbf{h}(n)$ for the channel impulse response $\mathbf{h}$. Since $\mathbf{h}$ is the singular vector corresponding to the smallest singular value of $\mathbf{W}$, following Lemma 1 and using Iteration (13), we propose

$$\mathbf{h}(n) = \frac{\mathbf{h}(n-1) - \nu(n)\mathbf{W}(n)\mathbf{h}(n-1)}{\|\mathbf{h}(n-1) - \nu(n)\mathbf{W}(n)\mathbf{h}(n-1)\|}, \qquad (15)$$

where $\nu(n) = 1/\text{trace}\{\mathbf{W}(n)\}$. In other words, at every time instant n and for every matrix $\mathbf{W}(n)$, we apply the second iteration of Lemma 1 *only once*.

It should be noted that whenever we are seeking only one singular vector (i.e. the case of the second subspace problem), then the orthonormalization step of (13) becomes a simple vector normalization. This special case of the orthogonal iteration is known in Numerical Analysis literature as the power method [5], [11].

Summarizing, under the assumption that an adaptive estimate of the noise subspace $\mathbf{U}_n$ is available, we first compute the matrix $\mathbf{W}(n)$ as in (14), and then apply (15) to obtain a new estimate for the channel impulse response. Let us now examine two possibilities for estimating $\mathbf{U}_n$.

A. Channel Estimation via RLS

Let $\mathbf{R}(n)$ be the exponentially windowed sample autocorrelation matrix of the data $\mathbf{y}(n)$, defined as $\mathbf{R}(n) = \sum_{i=0}^{n} \lambda^{n-i}\mathbf{y}(i)\mathbf{y}^H(i)$, with $0 < \lambda < 1$, a forgetting factor. If we denote by $\mathbf{P}(n)$ the inverse $\mathbf{P}(n) = \mathbf{R}^{-1}(n)$; then there exists the well known adaptation of the Recursive Least Squares (RLS) algorithm that computes directly $\mathbf{P}(n)$ from $\mathbf{P}(n-1)$ in $O((N+L)^2)$ operations.

With $\mathbf{P}(n)$ at hand we apply Lemma 1, using the first iteration, i.e. (12), as follows

$$\mathcal{S}(n) = \text{orthonorm}\{\mathbf{P}(n)\mathcal{S}(n-1)\}. \qquad (16)$$

Again we have used the same idea as in (15), that is, we have applied the iteration of Lemma 1 for the matrix $\mathbf{P}(n)$ only once. The proposed RLS scheme has the following complexity: the inverse data autocorrelation matrix $\mathbf{P}(n)$ requires $O((N+L)^2)$ operations; the multiplication in (16) $O((N+L)^2L)$, and the QR-decomposition again in (16) $O((N+L)L^2)$; the computation of $\mathbf{W}(n)$ in (14), by properly exploiting the Hankel structure of the matrix $\mathbf{V}_i(n)$, can be performed with $O((N+L)L^2)$ operations instead of $O((N+L)L^3)$; finally the adaptation of $\mathbf{h}(n)$ in (15), requires $O(L^2)$ operations. It is therefore clear that the total computational complexity is $O((N+L)^2L)$. Since usually $L \ll N$, the RLS version has computational complexity which is almost an order of magnitude smaller than Direct SVD techniques requiring $O((N+L)^3)$ operations.

B. Channel Estimation via LMS

Focusing always on Subspace Problem 1, let us consider an LMS like approach. We are going to use the well known idea that gave rise to the famous LMS algorithm; namely we use Lemma 1 and replace matrix $\mathbf{\Omega}$ of the second iteration (13) with the *instantaneous* estimate of the data autocorrelation matrix $\mathbf{R}$, i.e. the outer product of the received data $\mathbf{y}(n)\mathbf{y}^H(n)$. This results in the following adaptation.

$$\mathcal{S}(n) = \text{orthonorm}\Big\{\Big(\mathbf{I} - \frac{\mu}{\|\mathbf{y}(n)\|^2}\mathbf{y}(n)\mathbf{y}^H(n)\Big)\mathcal{S}(n-1)\Big\} \quad (17)$$

The complexity of this LMS scheme is as follows: all the matrix-vector operations in (17) demand $O((N+L)L)$, while the QR of (17), as well as (14) and (15) continue to have the same complexity as before, that is $O((N+L)L^2)$, $O((N+L)L^2)$ and $O(L^2)$ respectively. Here the most demanding parts are the QR-decomposition in (17) and the computation of $\mathbf{W}(n)$ (14). Thus, the overall complexity becomes $O((N+L)L^2)$. Compared to the RLS scheme, the LMS requires almost an order of magnitude less operations; not to mention that it is not necessary to guard in memory the large matrix $\mathbf{P}(n)$, as is the case in RLS.

V. Simulations

In this section, we provide several simulation results to demonstrate the performance of the blind adaptive channel estimation schemes developed herein. Let us briefly refer to the settings of the signaling scenario that we intend to follow. The number of orthogonal carriers comprising an OFDM block is set to $N = 64$, while the length of the ZP sequence to $L = 16$. The type of additive noise used in all simulations is Gaussian.

All graphs presented in the figures are the result of an average of 100 independent runs. In each run, we apply an abrupt change to the channel impulse response in order to examine the ability of the corresponding algorithms to follow the change. In particular we begin with $\mathbf{h}^T$=[0.5546 0.1604 0.1408 0.3161]+j[0.2138 0.6356 0.2904 -0.1138] and at time $n = 5000$ we switch to $\mathbf{h}^T$=[-0.1892 -0.2839 0.1274 -0.0451]+j[0.4273 0.6984 0.4321 0.0912]. The main characteristic of both channels is the fact that they strongly attenuate different frequency regions. More details can be found in [12].

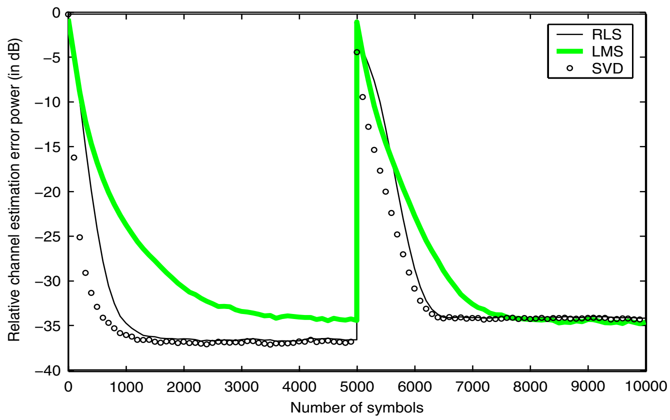

Fig. 2. Performance of RLS, LMS, and Direct SVD schemes; SNR=20 db.

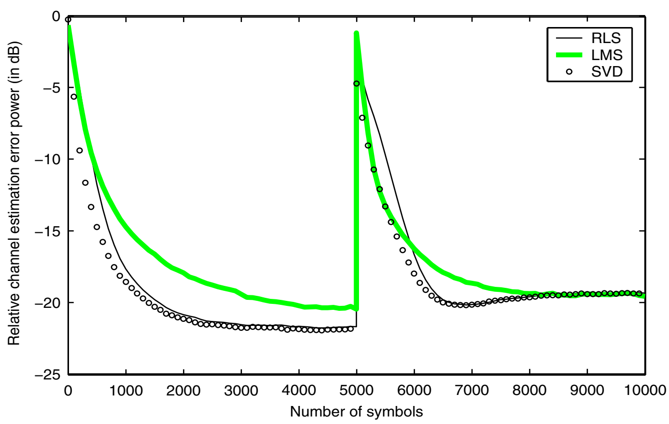

Fig. 3. Performance of RLS, LMS, and Direct SVD schemes; SNR=10 db.

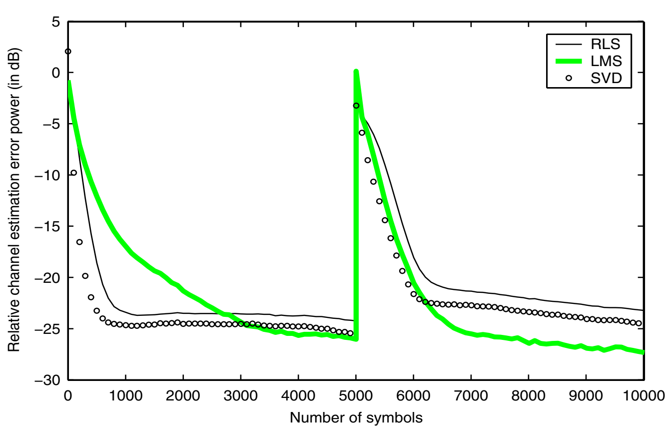

Fig. 4. Performance of RLS, LMS, and Direct SVD schemes in a slowly fading environment; SNR=15 db.

Although the channel lengths are equal to 4, we estimate them as being of maximum length $L + 1 = 17$.

In addition to the RLS and LMS versions, we simulate the Direct SVD approach. The latter consists in applying an SVD on $\mathbf{P}(n)$ to compute $\mathbf{U}_n(n)$ from which we obtain $\mathbf{W}(n)$, and then another SVD on $\mathbf{W}(n)$ to find $\mathbf{h}(n)$.

Fig. 2 depicts the relative channel estimation error of the three schemes for SNR equal to 20 dB; whereas Fig. 3 the corresponding performance for SNR=10 dB. In both cases, RLS matches closely the SVD approach, while LMS has a slightly inferior performance.

Finally Fig. 4 depicts the performance of our algorithms for SNR equal to 15 dB; only here both channels experience slow fading. As we can see, LMS can exhibit better performance than the other two schemes. This is evident from the fact that after time instant 5000, all schemes have similar convergence speed, but in steady state LMS attains a 5 dB lower power error level.

Summarizing, for all three SNR levels, both algorithms have a very satisfactory and consistent performance. They exhibit rapid convergence even after abrupt changes of the channel. RLS, despite its increased computational complexity, does not necessarily outperform LMS; in fact, this can be the case in fading channels where tracking is required.

VI. Conclusion

In this article, we have considered the problem of blind adaptive channel estimation in ZP-OFDM systems. With the help of two subspace problems we were able to obtain consistent estimates of the channel impulse response. Motivated by the orthogonal iteration, known from Numerical Analysis for the computation of singular vectors, RLS and LMS schemes were developed that provide adaptive channel estimates. Both algorithms are characterized by a simple repetitive structure with the LMS version having an attractively low computational complexity. The two algorithms exhibit rapid convergence and satisfactory steady state performance, being also capable of efficiently following both abrupt changes and slowly fading channels.

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