Prob. Theory and Math. Stat., Vol.1, pp. 119-122
Prohorov et al. (eds)
1986 VNU Science Press

DETECTION AND DIAGNOSIS OF CHANGES IN THE A.R. PART OF AN A.R.M.A. MODEL WITH NONSTATIONARY UNKNOWN M.A. COEFFICIENTS

M. Basseville, A. Benveniste, G. Moustakides, A. Rougée
I.R.I.S.A. (CNRS & INRIA)
Campus de Beaulieu
35042 RENNES Cedex - France

Problem statement :

$$\text{Let } y_t = \sum_{i=1}^{p} a_i \, y_{t-i} + \sum_{j=0}^{q} b_j(t)\, e_{t-j} \tag{1}$$

be a scalar ARMA(p,q) model, where (e_t) is a gaussian white noise with constant variance σ_e^2, and where the MA coefficients $(b_j(t))_j$ are time-varying, possibly subject to jumps. The problems to be solved are :

i) given a single (long) record of observations $(y_t^0)_{0\le t\le s}$, estimate the AR parameters, without using the unknown time-varying MA parameters ; then, calling "reference model" the AR part characterized by these estimated parameters,

ii) given another record of $(y_t^1)_{0\le t\le s}$, test whether the reference AR model still fit these new observations or has changed, again without knowing the (highly) time-varying MA coefficients ("global test") ;

iii) decide which poles have changed, i.e. solve the diagnosis problem.

Notations :

For $i = 0,1$, let $R_k^i(s)$ be the empirical covariances :

$R_k^i(s) = \sum_{t=0}^{s-k} y_{t+k}^i \; y_t^i \; (k \ge 0)$ and $\mathcal{H}_{p+1,N}^i(s)$ the (p+1)xN empirical Hankel matrices :

$$\mathcal{H}^{i}_{p+1,N} = \begin{pmatrix} R^{i}_{q-p+1}(s) & R^{i}_{q-p+2}(s) & \dots & R^{i}_{q-p+N}(s) \\ R^{i}_{q-p+2}(s) & & & \vdots \\ \vdots & & & \vdots \\ R^{i}_{q+1}(s) & \dots & \dots & R^{i}_{q+N}(s) \end{pmatrix}$$

where $N \geq p$.

<u>Theorem 1 |3|</u> :

The least squares solution $\theta \triangleq (a^{0}_{i}(s))_{1 \leq i \leq p}$ of the equation :

$$(-a_p \; - a_{p-1} \; \dots \; -a_1 \; 1) \, \mathcal{H}^{0}_{p+1,N}(s) = 0$$

is a *consistent* estimate of the true AR parameters, in the *nonstationary* case.

Precise conditions and proof (using the law of large numbers for martingales) can be found in |3|. In other words, the so-called instrumental variables method for identification is consistent even if the MA part is nonstationary, and thus solves problem i).

Let be : $U'_N(s) \triangleq (- a^{0}_{p} \; \dots \; - a^{0}_{1} \; 1) \, \mathcal{H}^{1}_{p+1,N}(s)$

$$= \sum_{t=q+N}^{s} w_t \, Z_t$$

where : $w_t = y^1_t - a^0_1 \, y^1_{t-1} - \dots - a^0_p \, y^1_{t-p}$ is the "moving average" part of the record (y^1_t) and $Z_t = (y^1_{t-q-1} \dots y^1_{t-q-N})'$. Let also be :

$$\hat{\Sigma}_N(s) = \sum_{t=q+N}^{s-q} \; \sum_{i=-q}^{q} w_t \, w_{t-i} \; Z_t \, Z'_{t-i} \; .$$

<u>Theorem 2 |5|</u> :

. *Nonstationary law of large numbers :* $\hat{\Sigma}_N$ is a *consistent* estimate of the true covariance matrix Σ_N of U_N, namely : $\Sigma^{-1}_N(s) \; \hat{\Sigma}_N(s) \xrightarrow{s \to \infty} I_N$, under both null hypothesis, i.e. the AR parameters are θ_0, and *local* alternative hypothesis, i.e. the parameters are $\theta_0 + \frac{\delta\theta}{\sqrt{s}}$, where $\delta\theta$ is fixed.

. *Central limit theorem :* $\Sigma_N(s)^{-1/2} \, . \, U_N(s) \xrightarrow{s \to \infty} \mathcal{N}(0, I_N)$ under $\mathbf{P}_{\theta_0}$

$$\hat{\Sigma}_N(s)^{-1/2} \cdot \left(U_N(s) - \mathcal{H}^{1'}_{p,N}(s) \cdot \frac{\delta\theta}{\sqrt{s}}\right) \xrightarrow{s\to\infty} \mathcal{N}(0, I_N) \text{ under } \mathbb{P}_{\theta_0 + \frac{\delta\theta}{\sqrt{s}}} .$$

Precise conditions and proof (limit theorems for martingales) can be found in |5|. In other words, the use of the *local* approach for detecting changes (|4|, |6|, |2|) reduces problem ii) to the problem of detecting a change in the mean of a Gaussian process. A possible χ^2 test for solving ii) is thus :

$$t_0 = U_N' \, \hat{\Sigma}_N^{-1} \, \mathcal{H}'_{p,N} \left(\mathcal{H}_{p,N} \, \hat{\Sigma}_N^{-1} \, \mathcal{H}'_{p,N}\right)^{-1} \mathcal{H}_{p,N} \, \hat{\Sigma}_N^{-1} \, U_N \qquad (2).$$

Let ψ be the set of the $m(\leq p)$ poles to be monitored, and J be the $p \times m$ Jacobian matrix $J_{p,m} = \left(\frac{\partial a_i}{\partial \psi_j}\right)_{\{\psi_j\} = \psi_0}$. Then the corresponding *diagnosis test* (solving problem iii) is of the form (2) where $\mathcal{H}'_{p,N}$ is replaced by : $\mathcal{H}'_{p,N} \, J_{p,m}$ (|1|). This approach allows, for example, separate monitoring of poles or subsets of poles, without knowing in advance which poles will actually change.

Extension to the vector case-applications to offshore platforms :

All these results are also *valid* in the *vector case*. See |3| and |8| for theoretical results. In |7| can be found numerical results (on real data) concerning the identification problem i) for vibrating characteristics of offshore platforms ; the main conclusion is that high order poles and modes are correctly identified. Results of simulations concerning the test ii) and diagnosis iii) problems in the scalar case can be found in |1| ; the main conclusion is that it is possible to detect and diagnose *small changes* (1 %) in eigenfrequencies, provided the sample size is large enough. Experimentations for the vector case are described in |9| and lead to discrimination between physically different changes.

References :

1. Basseville, M., Benveniste, A., Moustakides, G. (1986). Detection and diagnosis of abrupt changes in modal characteristics of non stationary digital signals. To appear in IEEE Trans. on Information Theory.

2. Basseville, M., Benveniste, A., Editors. (1986).Detection of abrupt changes in signals and dynamical systems. LNCIS n°77, Springer-Verlag, Berlin.

3. Benveniste, A., Fuchs, J.J. (1985). Single sample modal identification of a nonstationary stochastic process. IEEE Trans. on Aut. Contr., Vol. AC-30, n°1, pp 66-74.

4. Ibragimov, I.A., Khas'minskii, R.Z. (1972). Asymptotic behavior of statistical estimators in the smooth case. 1. study of the likelihood ratio. Th. of Proba. and Appl., Vol.17, n°3, pp 445-462.

5. Moustakides, G., Benveniste, A. (1986). Detecting changes in the AR parameters of a nonstationary ARMA process. To appear in Stochastics.

6. Nikiforov, I.V. (1983). Sequential detection of abrupt changesin time series properties. Moscow : Naouka.

7. Prevosto, M., Benveniste, A., Barnouin, B. (1982). Identification of vibrating structures subject to nonstationary excitation : a nonstationary stochastic realization problem. INRIA Research Report n°130.

8. Rougée, A. (1985). Détection de changements dans les paramètres AR d'un processus ARMA vectoriel : application à la surveillance des vibrations. Thèse de 3ème Cycle, Univ. de Rennes I.

9. Basseville, M., Benveniste, A., Moustakides, G., Rougée, A. (1985) Detection and diagnosis of changes in the eigenstructure of nonstationary multivariable systems. Submitted for publication.