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**DETECTION AND DIAGNOSIS
OF ABRUPT CHANGES
IN MODAL CHARACTERISTICS
OF NONSTATIONARY
DIGITAL SIGNALS**

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DETECTION AND DIAGNOSIS OF ABRUPT CHANGES IN MODAL CHARACTERISTICS OF NONSTATIONARY DIGITAL SIGNALS

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I - INTRODUCTION

The problem of detecting changes in spectral properties arises frequently in practice, in the segmentation of nonstationary digital signals or the monitoring of time series for example. Most of the solutions, which are available so far in the time domain, basically use the complete set of the (known or identified) model parameters : this is the case for innovations-based detectors and standard likelihood ratio tests. In [1] for example, a new on-line algorithm was presented, which was based upon Kullback's divergence between the conditional probability laws of the observations under a "reference" model and a "current" one.

In some practical situations, one is interested in detecting changes in a subset of the model parameters while the complementary subset of model parameters are completely unknown and thus have to be considered as nuisance parameters. This is the case for example in vibration monitoring where one wishes to detect changes in vibrating characteristics of systems subject to nonstationary unknown excitation such as swell, wind, earthquakes... In this case, the change detection problem can be formulated as follows : using an ARMA model with (highly) nonstationary unknown MA coefficients to model the excitation [9], detect a change in the AR part (assumed stationary) and if possible, determine which AR coefficients or which poles have changed (i.e. diagnosis problem).

Because of the highly time varying character of the unknown MA coefficients, none of the standard elimination methods for nuisance parameters [2] seems to be of help for this change detection problem. On the other hand, it has been shown in [3] that it is possible to obtain consistent estimates of the AR coefficients without knowing (or using estimates of) the varying MA coefficients.

The basic idea underlying this paper is thus the following : instead of using standard likelihood ratio methods which are of no help in the present case because of the unknown MA part, one may base the detection upon the same idea which is used for the identification. Using this idea a statistics U is introduced and the central limit theorem is shown to hold [6] for this

statistics U under both hypotheses, null H_0 (i.e. no change) and local alternative H_1 (i.e. small change). This gives a test statistics for a global test (for a change in the AR part) with no diagnosis about the nature of the change. Using the effect of specific parameter changes, such as for example changes in poles or vibrating modes, on the mean of U under H_1 , one may design specific tests for monitoring vibrating modes separately. Another approach for solving the diagnosis problem will be presented : it consists in basically reidentifying each pole which has to be monitored.

In this paper, we investigate only the scalar case. The extension of the proposed test to the vector case will be reported later ; let us only emphasize that this extension may be used to solve the problem of vibration monitoring for offshore platforms for example.

Sections II and III will be devoted to the presentation of the proposed off-line algorithms for the two problems of interest : *detection* and *diagnosis*, while numerical results are given in section IV. The performances in the simple case of changes in AR models (no MA part) are investigated in paragraph IV.1, with special emphasis on the problem of coupling effects that arises during diagnosis upon the poles.

The performance of the proposed tests in simulations with scalar ARMA signals is reported in paragraph IV.2, for the case where the nonstationary MA part is piecewise constant.

Section V outlines the main conclusions of this study.

II - DETECTION OF CHANGES IN THE AR PART OF AN ARMA MODEL WITH NONSTATIONARY UNKNOWN MA COEFFICIENTS

As mentioned in the INTRODUCTION, this paper is focussed on the scalar case. Let us thus consider the following model :

$$y_t = \sum_{i=1}^p a_i y_{t-i} + \sum_{j=0}^q b_j(t) e_{t-j} \quad (1)$$

where $(e_t)_t$ is a gaussian white noise with constant variance σ_e^2 .

Here the unknown moving average coefficients (b_j) are time varying, and may even be subject to jumps. The problem to be solved is the (off-line) detection of abrupt changes or jumps in the autoregressive parameters (a_i) . We shall first recall the main result concerning the identification problem, because as mentioned in the INTRODUCTION, it is the starting point of our detection procedure.

1. - Identification of the AR coefficients without knowing the non-stationary MA coefficients

Assume that a single record $(y_1, \dots, y_s)$ of the process (y_t) is available. The so-called instrumental variable method for identification has been recently proved [3] to provide with consistent estimates of the autoregressive parameters in the present framework. More precisely, let :

$$\mathcal{H}_{p,N-1}(s) = \begin{bmatrix} R_{q-p+1}(s) & R_{q-p+2}(s) & & R_{q+1}(s) & R_{q+2}(s) & \dots & R_{N+q-p}(s) \\ R_{q-p+2}(s) & & & & & & \\ R_{q-p+3}(s) & & & & & & \\ \vdots & & & & & & \\ R_{q+1}(s) & R_{q+2}(s) & & & & & \\ & & & & & & R_{N+q}(s) \end{bmatrix} \quad (2)$$

the $(p+1) \times N$ empirical Hankel matrix of the process $(y_t)_{0 \leq t \leq s}$ where

$N \gg p$ is the number of instruments and

$$R_k(s) = \sum_{t=0}^{s-k} y_{t+k} y_t \quad (k \geq 0) \quad .$$

Then the least squares solution $(\hat{a}_p(s), \hat{a}_{p-1}(s), \dots, \hat{a}_1(s))$ of the equation :

$$(-a_p \ -a_{p-1} \ \dots \ -a_1 \ 1) \mathcal{H}_{p,N-1}(s) = 0$$

is a consistent estimate of true vector parameter :

$$\theta = (a_p \ a_{p-1} \dots a_1)' \quad (3)$$

of model (1). See [3] for a complete proof and precise statement of the consistency result. This result does not require any stationarity assumption about the moving average parameters $b_j(t)$. In this sense, this identification method of the AR part may be thought as being robust with respect to the unknown MA part.

2. - The change detection problem

The use of standard observation-based likelihood ratio techniques for solving this problem would require either an identification of the MA coefficients $b_j(t)$ using for example a forgetting factor, or maximization or integration of the likelihood with respect to a prior distribution of these unknown parameters [2]. Because of their highly varying features (related for example to the shock or turbulence effects of the sea on an offshore platform), these approaches do not seem to be appropriate. (Remember also that in [4] Bohlin assumed that convenient values of the MA coefficients were available).

Moreover, the Fisher information matrix of an ARMA model is not block diagonal : there is an interaction between the AR and the MA coefficients. In other words, there is a coupling effect between the detection of changes in poles or zeroes, and therefore it is not convenient to use (local) likelihood methods ([5], [7], [12]) for detecting changes on poles when the zeroes have to be viewed as nuisance parameters.

Keeping in mind the "robustness" properties of the identification procedure with respect to the nuisance parameters, we propose the following off-line change detection procedure. Let us now assume that a "reference" model parameter $\theta_0 = (a_p^0 \dots a_1^0)'$ has been estimated on a record of signals y , and let us consider the following problem : given a new record of signals y , decide whether they follow the same model or not. The solution we propose is the following : compute again the empirical Hankel matrix $\mathcal{H}_{p,N}$ corresponding to this new record, and look at the "size" of the vector u_N

defined by : $U_N'(s) = (-a_p^0 \dots -a_1^0 \ 1) \mathcal{H}_{p,N-1}(s)$ (4)

If there has been no change in the AR part, this U vector should be "close" to zero ; in case though of a change in the AR parameters, this U vector should be "significantly" different from zero.

Let us rewrite $U_N(s)$, in a numerically more efficient way, as :

$$U_N(s) = \sum_{t=q+N}^s w_t Z_t \quad (5)$$

where : $w_t = y_t - a_1 y_{t-1} - \dots - a_p y_{t-p}$ (6)

is the "moving average part", and :

$$Z_t = (y_{t-q-1} \ y_{t-q-2} \ \dots y_{t-q-N})' \quad (7)$$

Under the hypothesis of no change (i.e. θ_0 still represents the AR part of the actual process), Z_t is *orthogonal* to w_t and the covariance matrix of U is :

$$\hat{\Sigma}_N(s) = \sum_{t=q+N}^{s-q} \sum_{i=-q}^q \mathbb{E}_{\theta_0} (w_t w_{t-i} Z_t Z_{t-i}') \quad (8)$$

because, for $|t-r| \geq q+1$: $\mathbb{E}_{\theta_0} (w_t w_r Z_t Z_r') = 0$.

Finally, let $\hat{\Sigma}_N(s)$ be the following matrix :

$$\hat{\Sigma}_N(s) = \sum_{t=q+N}^{s-q} \sum_{i=-q}^q w_t w_{t-i} Z_t Z_{t-i}' \quad (9)$$

Despite the fact that the process y_t , and thus Z_t , is *nonstationary*, it turns out that the two following theorems hold [6] :

i) *Nonstationary law of large numbers*

$\hat{\Sigma}_N$ is a consistent estimate of Σ_N , namely :

$$\Sigma_N^{-1}(s) \hat{\Sigma}_N(s) \xrightarrow{s \rightarrow \infty} I_N \quad (10)$$

under both null hypothesis, i.e. the set of AR parameters is θ_0 , and local alternative hypothesis, i.e. the set of AR parameters is $\theta_0 + \frac{\delta\theta}{\sqrt{s}}$, where $\delta\theta$ is fixed.

ii) *Central limit theorem*

Under the probability law $\mathbb{P}_{\theta_0}$, we have :

$$\hat{\Sigma}_N(s)^{-\frac{1}{2}} \cdot U_N(s) \xrightarrow{s \rightarrow \infty} \mathcal{N}(0, I_N) \quad (11)$$

and under the "small" change hypothesis $\mathbb{P}_{\theta_0 + \frac{\delta\theta}{\sqrt{s}}}$ we have :

$$\hat{\Sigma}_N(s)^{-\frac{1}{2}} \cdot (U_N(s) - \mathcal{H}_{N-1,p-1} \frac{\delta\theta}{\sqrt{s}}) \xrightarrow{s \rightarrow \infty} \mathcal{N}(0, I_N) \quad (12)$$

Basically, the proofs are based upon extensive uses of various limit theorems for martingales.

In other words, the use of the local approach for detecting changes (Ibragimov and Khash'minski [5], Nikiforov [7], [12]), reduces the original problem to the problem of detecting a change in the mean value of a gaussian process.

Let us consider the generalized likelihood ratio [11] as the decision rule for this new problem. Maximization with respect to all possible "magnitudes" of changes $\delta\theta$ is straightforward, and leads to the following χ^2 test :

$$t_0 = U_N' \hat{\Sigma}_N^{-1} \mathcal{H}_{N-1,p-1} \left(\mathcal{H}_{p-1,N-1} \hat{\Sigma}_N^{-1} \mathcal{H}_{N-1,p-1} \right)^{-1} \mathcal{H}_{p-1,N-1} \hat{\Sigma}_N^{-1} U_N \quad (13)$$

3. - More on some special cases

a) Let us first investigate the AR case where $q=0$.

$$\text{Then : } U_N(s) = \sum_{t=N}^S w_t Z_t$$

$$\text{where } w_t = b_0(t) e_t$$

$$\text{and } Z_t = (y_{t-1} \dots y_{t-N})' \quad (14)$$

Using the formulation with the empirical Hankel matrix :

$$U_N(s) = \begin{bmatrix} R_{-p+1}(s) & R_{-p+2}(s) & \dots & R_0(s) & R_1(s) \\ R_{-p+2}(s) & & & & R_2(s) \\ \vdots & & & & \vdots \\ R_0(s) & & & & \vdots \\ \vdots & & & & \vdots \\ R_{N-p}(s) & \dots & \dots & \dots & R_N(s) \end{bmatrix} \begin{bmatrix} -a_p \\ -a_{p-1} \\ \vdots \\ -a_1 \\ 1 \end{bmatrix} \quad (15)$$

(start from R_{-p+1}) .

$$\begin{aligned} \text{Furthermore : } \sum_{t=N}^S \mathbb{E}_{\theta_0} (w_t^2 Z_t Z_t') \\ &= \sum_{t=N}^S \mathbb{E}_{\theta_0} (w_t^2) \mathbb{E}_{\theta_0} (Z_t Z_t') \\ &= \sigma_e^2 \sum_{t=N}^S b_0^2(t) \mathbb{E}_{\theta_0} (Z_t Z_t') \end{aligned}$$

where $\mathbb{E}_{\theta_0} (Z_t Z_t')$ is the Toeplitz covariance matrix of size N of the observation process $(y_t)_t$.

$\hat{\Sigma}_N$ may be estimated by :

$$\hat{\Sigma}_N(s) = \sum_{t=N}^S w_t^2 z_t z_t' \quad (16)$$

Let us now consider the *stationary* AR case, i.e. let us assume that $b_0(t)$ is constant, which is usually the case when one is interested in changes in AR parameters. Then :

$$\Sigma_N(s) = b_0^2 \sigma_e^2 \sum_{t=N}^S \mathbb{E}_{\theta_0} (z_t z_t') \quad (16)$$

may be estimated by :

$$\hat{\Sigma}_N(s) = b_0^2 \sigma_e^2 \sum_{t=N}^S z_t z_t' \quad (17)$$

Furthermore let us assume that $N = p$ which is the minimum number of instruments to be used. Then the empirical Hankel matrix $\hat{\Sigma}_{p-1,p-1}$ is generically invertible, and the global test t_0 (13) is :

$$t_0 = U_p' \hat{\Sigma}_p^{-1} U_p \quad (18)$$

which is nothing but the classical local likelihood ratio test (first version of the cusum type algorithm derived by I.V. NIKIFOROV for detecting changes in AR coefficients [8], [12]).

b) Let us now consider the ARMA (p,p-1) case which naturally arises from state space models without observation noise ; this model has been used for the vibration monitoring application on offshore platforms [9]. Then

from (4) and (2) :

$$U_N(s) = \begin{bmatrix} R_0(s) & R_1(s) & \cdots & R_{p-1}(s) & R_p(s) \\ R_1(s) & & & & R_{p+1}(s) \\ \vdots & & & & \vdots \\ R_{p-1}(s) & & & & \vdots \\ \vdots & & & & \vdots \\ R_{N-1}(s) & \cdots & \cdots & \cdots & R_{N+p-1}(s) \end{bmatrix} \begin{bmatrix} -a_p \\ -a_{p-1} \\ \vdots \\ -a_1 \\ 1 \end{bmatrix}$$

(start from R_0) ,

and from (5) :

$$U_N(s) = \sum_{t=N+p}^S w_t Z_t$$

with :

$$w_t = y_t - a_1 y_{t-1} \cdots -a_p y_{t-p}$$

$$Z_t = (y_{t-p} \ y_{t-p-1} \ \cdots \ y_{t-N-p+1})'$$

For $N = p$, the global test t_0 (13) is still as in (18).

It has to be noticed that, instead of the estimate (9) of $\sum_N$:

$$\hat{\sum}_N(s) = \sum_{t=N+p}^S \sum_{i=-p+1}^{p-1} w_t w_{t-i} Z_t Z_{t-i}' ,$$

another "approximate" estimate may be computed, which is :

$$\tilde{\sum}_N(s) = \left(\sum_{t=p+1}^S w_t^2 \right) \left(\sum_{t=N+p}^S Z_t Z_t' \right) \quad (19)$$

(see (17)), and which turns out to lead to a global test numerically better conditioned than the initial one, although we have no theoretical justification for it.

Finally, let us mention that all these tests may be extended to the vector case. This will be reported later.

III - DETECTION WITH DIAGNOSIS

Here we investigate the problem of detecting changes in the AR part, with diagnosis upon which AR coefficients or which poles have changed, and still without knowing the nonstationary MA coefficients. As in section II, we only investigate the scalar case. Let us first emphasize that, even in the stationary AR case, this diagnosis problem is not so much standard, especially when the poles are of interest. As far as we know, the only approach which has been investigated for solving this problem is the so-called multiple model approach, described for example in [10], [11]. Two approaches are presented in this section : a sensitivity method which looks for changes (on the AR parameters or on the poles) constrained into a subspace, and a decoupling method which is a kind of filter bank approach and which basically re-identifies each pole to be monitored.

1. - Sensitivity method

It has been shown in the previous section that a possible solution to the problem of detecting changes in the AR parameters without knowing the MA ones, is to solve the equivalent gaussian testing problem for the "instrumental" statistics U . Remember (11) and (12) which summarize the nonstationary central limit theorem.

The basic idea underlying the sensitivity method is to take into account the effect $\delta\theta$ of changes of interest (for example on separate poles) on the θ parameter (3) and to use the same likelihood ratio approach based upon the U vector.

We will make an extensive use of the following general result. Assume U is (asymptotically) distributed as $\mathcal{N}(0, \Sigma)$ under H_0 and as $\mathcal{N}(\mu, \Sigma)$ under H_1 . For testing $\mu = 0$ against $\mu \neq 0$, one has to compute :

$$U' \sum^{-1} U \quad (20)$$

For testing $\mu = 0$ against $\mu \in \text{Range}(A)$, where A is a given low rank matrix, compute :

$$U' \sum^{-1} A (A' \sum^{-1} A)^{-1} A' \sum^{-1} U \quad (21)$$

which is nothing but the maximum value, with respect to ν , of the log likelihood ratio between H_0 and H_1 with $\mu = A\nu$.

Describing more precisely the diagnosis problem, let ψ be the m -dimensional set of the "free" parameters to be monitored, and ψ_0 the set of their nominal values. Then, changes $\delta\psi$ in these free parameters induce changes in the AR parameters $\delta\theta$ given by :

$$\delta\theta = f(\delta\psi)$$

where f is a non linear differentiable function.

Let $J = f'(\psi_0)$ be the pxm Jacobian matrix :

$$J = \left(\frac{\partial a_i}{\partial \psi_j} \right)_{\{\psi_j\} = \psi_0} \quad (22)$$

A first order approximation leads to :

$$\delta\theta = J \delta\psi ;$$

in other words the changes on the AR parameters are constrained to the subspace $\text{Range}(J)$. The corresponding diagnosis test is nothing but (21) with

$$A = \begin{bmatrix} \mathcal{H}_{N-1, p-1} & J_{p, m} \end{bmatrix}$$

For example, if the diagnosis problem of interest is to monitor eigenfrequencies ω_j , the corresponding tests are :

$$G_j' W_j^{-1} G_j \approx \lambda \quad (23)$$

where

$$\begin{cases} G_j = \frac{\partial \theta}{\partial \omega_j} H' \Sigma^{-1} U \\ W_j = \frac{\partial \theta}{\partial \omega_j} H' \Sigma^{-1} H \frac{\partial \theta}{\partial \omega_j} \end{cases} \quad (24)$$

and where $\frac{\partial \theta}{\partial \omega_j}$ may be computed off-line (numerical values of the derivatives are computed at the nominal poles). The computation of the Jacobian matrix J in this special case may be found in the Appendix.

The advantage of this approach is that it allows the separate monitoring of as many poles or subsets of poles as desired, without a priori knowing which poles will actually change. The main drawback is that there may exist a coupling effect between the poles to be monitored ; namely all the separate tests can be non zero even if only one pole has actually moved. But it will be shown in section IV that the diagnosis decision is nevertheless correct, in most of the cases.

2. - Decoupling method

The basic idea of this approach is to re-identify the poles which have to be monitored, and to use the global tests (13) or (18) associated to the "small order" corresponding U vectors. For simplicity, let us consider the case where no pole is real, and thus $p = 2r$.

$$\text{Let be : } P(z) = z^{2r} - \sum_{i=1}^{2r} a_i z^{2r-i}$$

the characteristic polynomial of the model, and let us consider all the possible factorizations of the form :

$$P(z) = (z - \lambda_j) (z - \bar{\lambda}_j) P_j(z) ..$$

The decoupling method for diagnosis is depicted on figure n°1 and is as follows. For each index j of interest, achieve the inverse filtering of the signal (y_t) through P_j . On the resulting signal, identify the AR part of an ARMA $(2,q)$ model, in the same manner as in section II 1) for example. Then, using the new "nominal" values a_1^j and a_2^j compute the corresponding U^j vector via (4) and the χ^2 test (18) :

$$T^j = U^{j'} \hat{\Sigma}_j^{-1} U^j$$

with $p = 2$ and with $\hat{\Sigma}$ estimated via (9) or (19).

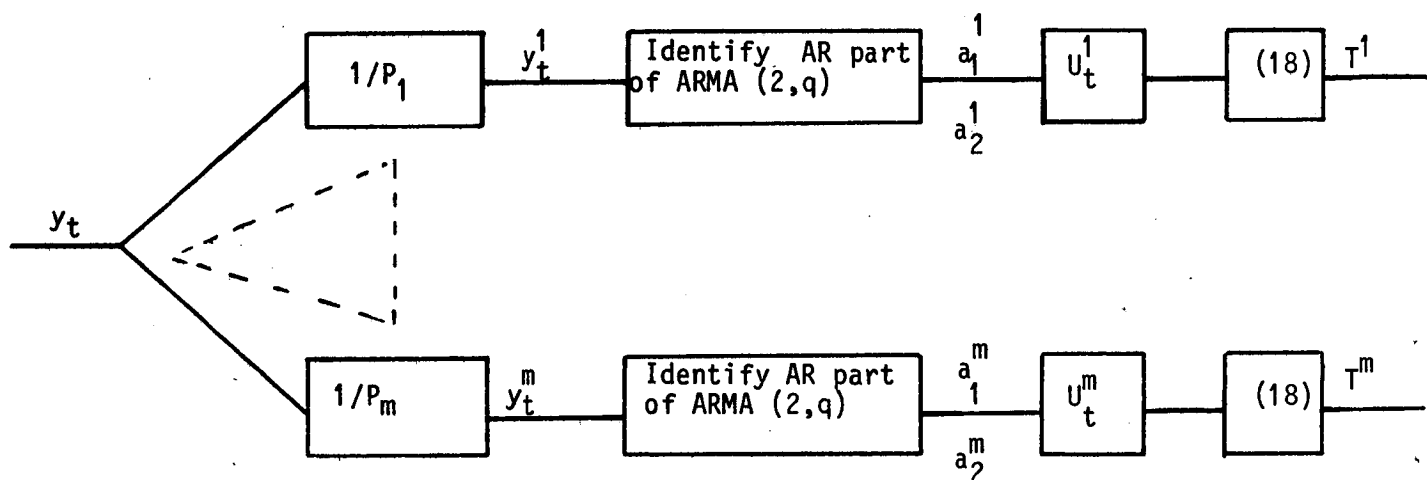


Figure n°1

If there is no change at all, all the T^j tests will be zero. If there is a change on the pole λ_i , then the test T^i is zero while all the other ones (T^j for $j \neq i$) are non zero.

The obvious advantage of this method is that there is no more coupling effect between the poles : if only one pole is moving, only one test is closed to the "good" value (here zero). The main drawback of this method is that if $m \geq 2$ poles are moving simultaneously, then we need to perform a number of tests that is equal to the number of combinations of m elements

among n (order of the system). The sensitivity method requires only m tests. Furthermore, the decoupling method requires in practice the prior knowledge of which subsets of poles are moving, otherwise the decoupling property is lost.

Numerical results concerning these two methods for diagnosis will be given in the next section. Let us only outline that their extension to the vector case is possible and currently under study.

IV - NUMERICAL RESULTS

We here investigate the numerical behavior of the tests which we have presented in the two previous sections. The main points to be emphasized are that :

- i) the global test (18) is an efficient approach, especially when the estimated $\hat{\Sigma}$ matrix is computed via (19) ;
- ii) the sensitivity method, despite its coupling effect, is able to detect and diagnose small changes in eigenfrequencies ;
- iii) the decoupling method is efficient for diagnosis, when only one pole moves.

The experiments which have been done are highly motivated by the fact that, in view of the application to vibration monitoring, we are interested in detecting *small* changes in eigenfrequencies. "Small" here means one percent ; in other words, according to the location of the corresponding poles, the "observable" change, namely the change in the AR coefficients, may be less than four per thousand.

We have chosen models of even order, with pairs of complex conjugate poles, of the form :

$$(\rho_j e^{i\omega_j}, \rho_j e^{-i\omega_j})$$

and studied changes in one or more ω_j . In most cases, the ρ_j are equal, but the influence of these parameters has also been studied. We will show

that a fixed pole close to the unit circle can prevent the diagnosis, and even the "global" detection of a change in a second pole far from the unit circle. The models which have been used are shown in table 1. For each experiment, the numerical values of the test statistics are computed under both H_0 (i.e. the actual model is the reference model) and H_1 (i.e. the actual model is the changed model).

Table 2 gives the values of the global test and the sensitivity test for diagnosis in the special case of *stationary AR signals*. The reason for considering this case is the analysis of the coupling effect during diagnosis mentioned in section III.1).

Table 3 gives the corresponding results for the ARMA (p,p-1) case, where the moving average coefficients are piecewise constant (lengths of intervals randomly chosen).

It has to be emphasized that the numerical values given in these tables are pessimistic because they give the mean values obtained by mixing experiments with different record lengths (from 1000 to 10000, by increments of 500), and in many experiments it is obvious that the behavior of the tests especially for the diagnosis is very poor when less than 3000 or 4000 sample points are available (remember the small magnitude of the changes under study).

V - CONCLUSION

The problem of detection and diagnosis of changes in modal characteristics of nonstationary (scalar) digital signals has been addressed. An equivalent problem is to detect changes in the poles of an ARMA model having nonstationary unknown moving average coefficients. New tests have been derived, and studied via a simulation study. The main idea underlying our approach is to use a likelihood ratio technique, but based upon an "instrumental" statistics (rather than the observations themselves) which is more "robust" with respect to the nuisance parameters. The main conclusion is that detection and diagnosis of small (1%) changes in eigenfrequencies are possible, provided that the size of the sample is large enough (several

thousands) and that there is no "masking effect", namely that the poles to be monitored are not less close to the unit circle than other ones. This latter point is to be further investigated theoretically. Finally, the extension of these tests to the case of *vector* signals is possible and currently under study.

LIST OF TABLES

Table I : *Nominal and changed models used for the simulation study.*

Table II : Investigation of the coupling effect of the sensitivity method in the *stationary* AR case.

Table III : Diagnosis by sensitivity and decoupling methods in the *non-stationary* ARMA case (piecewise constant MA coefficients).

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Table I

	name of Experiment	Reference model		Changed model	Relative magnitude/ ω	Relative "observable" magnitude
		ρ_j	ω_j	ω_j		
A R 4	e41	0.99 0.99	1.9 0.5	1.9 0.495	1%	3°/∞
	e42	0.99 0.99	0.8 0.6	0.8 0.594	1%	4°/∞
	e43	0.99 0.99	1.5 0.5	1.5 0.495	1%	3°/∞
	e44	0.99 0.99	2.2 2.4	2.18 2.4	1%	3°/∞
	e45	0.98 0.99	1.9 0.5	1.9 0.495	1%	3°/∞
	e46	0.99 0.98	1.9 0.5	1.9 0.495	1%	3°/∞
	e47	0.98 0.99	0.8 0.6	0.8 0.594	1%	4°/∞
	e48	0.99 0.98	0.8 0.6	0.8 0.594	1%	4°/∞
	e49	0.99 0.6	0.8 0.6	0.8 0.594	1%	4°/∞
A R 6	e61	0.99 0.99 0.99	1.9 0.6 0.4	1.9 0.6 0.396	1%	2°/∞
	e62	0.99 0.99 0.99	1.5 0.6 0.4	1.5 0.6 0.396	1%	2°/∞
	e63	0.99 0.99 0.99	1.5 0.6 0.4	1.5 0.594 0.4	1%	4°/∞
	e64	0.99 0.99 0.99	0.8 0.6 0.4	0.8 0.6 0.396	1%	2°/∞
	e65	0.99 0.99 0.99	0.8 0.6 0.4	0.8 0.594 0.4	1%	4°/∞

Table II

Exp.	H_0		H_1	
	Global test	Sensitivity test	Global test	Sensitivity test
e41	3.78	0.93 0.60	15.14	0.85 14.15
e42	3.32	1.09 0.93	17.99	1.24 17.90
e43	2.95	0.90 1.25	13.68	1.57 11.81
e44	4.42	1.37 1.06	175.69	189.32 1.18
e45	3.44	0.99 0.69	14.12	0.87 13.57
e46	3.93	0.65 0.61	9.05	1.44 7.69
e47	4.39	1.13 0.87	18.21	1.27 15.94
e48	3.15	0.61 1.09	10.33	1.30 7.79
e49	4.52	1.13 1.87	4.12	0.90 1.01
e61	4.99	0.35 1.09 0.66	13.23	1.27 1.30 7.55
e.62	6.54	0.61 1.36 0.87	13.32	1.82 1.09 10.96
e63	6.54	0.61 1.36 0.87	22.35	0.83 16.74 1.29
e64	7.01	1.07 1.39 1.74	17.51	1.96 1.14 13.50
e65	7.01	1.07 1.39 1.74	22.10	1.74 17.43 1.68

Table III

Exp.	H_0			H_1		
	Global test	Sensitivity test	Decoupling test	Global test	Sensitivity test	Decoupling test
e41	2.56	0.89 0.49	2.39 2.00	14.90	0.43 13.20	15.15 1.81
e42	1.75	0.48 0.59	0.75 0.69	11.29	0.84 9.27	22.74 0.75
e43	2.57	1.08 0.49	1.97 0.93	7.23	0.29 6.33	17.63 0.82
e44	4.29	0.70 1.54	7.55 4.86	237.35	233.03 1.23	6.22 312.29
e61	2.27	1.17 0.26 0.61	4.14 1.72 1.84	9.84	0.44 0.30 6.70	13.24 11.10 3.80
e62	2.69	0.54 0.30 0.36	5.09 6.10 2.66	7.27	0.92 0.26 3.93	9.21 7.44 3.64
e64	4.11	0.17 0.25 0.62	5.53 4.09 6.20	8.99	0.84 0.47 4.21	20.60 13.72 3.63

APPENDIX

COMPUTATION OF THE JACOBIAN MATRIX

Let $\theta = (a_p \ a_{p-1} \ \dots \ a_1)'$ the vector of the AR parameters of the ARMA model to be tested. Assume that the AR order is even, and let the poles be of the form :

$$\lambda_j = \rho_j e^{\pm i\omega_j}$$

where the eigenfrequency ω_j is such that $0 < \omega_j < \pi$. The purpose of this Appendix is to give the derivation of the Jacobian matrix $(\frac{\partial \theta}{\partial \omega_j})$ which is needed for the diagnosis problem when the sensitivity method is used. See formula (22).

The p poles λ_j satisfy the following characteristic equation :

$$\sum_{k=0}^p a_k \lambda^{p-k} = 0$$

where $a_0 = -1$. Computing the derivatives of these relations with respect to one eigenfrequency to be monitored, say ω_{j_0} , we get the following system of p equations with p unknown variables :

$$\left\{ \begin{array}{l} i \sum_{k=0}^{p-1} a_k (p-k) \lambda_{j_0}^{p-k} + \sum_{k=1}^p \frac{\partial a_k}{\partial \omega_{j_0}} \lambda_{j_0}^{p-k} = 0 \\ -i \sum_{k=0}^{p-1} a_k (p-k) \overline{\lambda_{j_0}}^{p-k} + \sum_{k=1}^p \frac{\partial a_k}{\partial \omega_{j_0}} \overline{\lambda_{j_0}}^{p-k} = 0 \\ \text{for } j \text{ such that } \lambda_j \neq \begin{cases} \lambda_{j_0} \\ \overline{\lambda_{j_0}} \end{cases} : \sum_{k=1}^p \frac{\partial a_k}{\partial \omega_{j_0}} \lambda_j^{p-k} = 0 \end{array} \right.$$

In other words :

$$\psi \frac{\partial \theta}{\partial \omega_j} = -i \begin{bmatrix} 0 \\ \vdots \\ 0 \\ \beta_j \\ 0 \\ \vdots \\ 0 \\ -\bar{\beta}_j \\ 0 \\ \vdots \\ 0 \end{bmatrix} \begin{matrix} \swarrow \\ \searrow \end{matrix} \begin{matrix} \text{lines number } j \text{ and } (j + \frac{p}{2}) \end{matrix}$$

where : $\beta_j = \sum_{k=0}^{p-1} a_k (p-k) \lambda_j^{p-k}$

and ψ is the Van der Monde matrix :

$$\psi = \begin{bmatrix} 1 & \lambda_1 & \lambda_1^2 & \dots & \lambda_1^{p-1} \\ \vdots & \vdots & \vdots & & \vdots \\ 1 & \lambda_{p/2} & \lambda_{p/2}^2 & \dots & \lambda_{p/2}^{p-1} \\ \vdots & \vdots & \vdots & & \vdots \\ 1 & \bar{\lambda}_1 & \bar{\lambda}_1^2 & \dots & \bar{\lambda}_1^{p-1} \\ \vdots & \vdots & \vdots & & \vdots \\ 1 & \bar{\lambda}_{p/2} & \bar{\lambda}_{p/2}^2 & \dots & \bar{\lambda}_{p/2}^{p-1} \end{bmatrix}$$

Thus, if u_j and v_j are the columns of ψ^{-1} , with numbers j and $(j + \frac{p}{2})$ respectively, we have :

$$\frac{\partial \theta}{\partial \omega_j} = i (-\beta_j u_j + \bar{\beta}_j v_j) .$$

Actually, it is not difficult to prove that $v_j = \bar{u}_j$. So :

$$\frac{\partial \theta}{\partial \omega_j} = 2 \operatorname{Im} (\beta_j u_j) .$$

For each pole to be monitored, the only values to be computed are thus the complex number β_j and the appropriate column of the inverse of the matrix ψ . Using the lemma for inverting a partitioned matrix, one can easily replace the inversion of the $(p \times p)$ matrix ψ by two inversions of $(p/2 \times p/2)$ matrices and products of matrices of this size.

Another way of computing the inverse of ψ without inverting any matrix is to use the following remark : the column number j of ψ^{-1} , say $(c_0^j, c_1^j, \dots, c_{p-1}^j)$, satisfies :

$$c_0^j + c_1^j \lambda_i + \dots + c_{p-1}^j \lambda_i^{p-1} = \begin{cases} 0 & \text{for all } i \neq j \\ 1 & \text{for all } i = j \end{cases}.$$

Thus, the ratios $\frac{c_k^j}{c_{p-1}^j}$ ($0 \leq k \leq p-2$) are the coefficients of the

polynomial $\prod_{i \neq j} (s - \lambda_i)$, and c_{p-1}^j is determined by using the last relation.

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