Proceedings of 25th Conference
on Decision and Control
Athens, Greece • December 1986

OPTIMAL SENSOR LOCATION FOR DETECTING CHANGES IN DYNAMICAL BEHAVIOR

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Abstract.- We address the problem of optimal sensor location for monitoring the eigenstructure of a multivariable dynamical system. The criterions which are optimized are the power of new tests designed for detecting and diagnosing changes in the eigencharacteristics of a system |3| |12|. The key points are the choice of the parametrization for computing the criterion and the comparison of designs with different number of sensors. The discussion of the numerical results for sensor location includes the analysis of the effect of the geometry of the unknown excitation.

Key words : Sensor location, Change detection and diagnosis, Power of statistical tests.

INTRODUCTION

The problem of optimal sensor location is of crucial importance in system design, in order to reduce the cost of instrumentation and increase the efficiency of the identifiers, state estimators or detectors which are implemented for monitoring the system. Except for distributed parameter systems, this problem seems to have received a little attention in the literature. Furthermore, it has been addressed more in view of good parameter estimation or state reconstruction, than for optimum monitoring of the system parameters and for change or failure detection.

1. SHORT REVIEW OF EXISTING RESULTS

The results which are available so far, up to our knowledge, may be classified as follows:

1.1 Optimal sensor location for state reconstruction

The problem is to find a measurement matrix H which optimizes a criterion reflecting the performances of the optimum state estimator (or smoother) for the considered linear dynamical system. Various criterions have been investigated. Using a direct sensitivity analysis of the state estimate covariance matrix P, |1| suggested to solve the nonlinear programming problem:

$$\min_{H} \operatorname{tr} (W \dot{P})$$

where W is a weighting matrix.

Several other authors |4| |7| |8| used indirect measures of performance Γ , such as Fisher information matrix (for state estimation) or observability matrix. Furthermore, several **scalar** performance indexes related to Γ were studied. The most general one is |8| |7| :

$$m_s = (\operatorname{tr} \Gamma^s/n)^{1/s} \quad , s \leqslant 0, \; n = \text{state dim}$$

which, in some special cases, reduces to the following widely used criterions :

$$\lim_{s\to 0} m_s = |\Gamma|^{1/n} \qquad \text{(determinant norm)}$$

$$m_{-1} = n/\operatorname{tr}(\Gamma^{-1}) \qquad \text{(trace norm)}$$

$$m_{-\infty} = \lambda_{min}(\Gamma) \qquad \text{(extremal eigenvalue)}$$

Finally, the case of non-linear systems was studies in |5|, with the aid of the trace norm for the Fisher information matrix of the unknown parameters to be estimated.

One common feature of all these investigations is that the measurement matrix H is supposed to be of "continuous" type, namely to have real (and not integer) coefficients.

The dual problem of optimum controller location was investigated in |8|, with the aid of the above mentioned general norm for the controllability matrix.

1.2 Optimum sensor and controller location for distributed parameter systems

These problems have been widely investigated for the last fifteen years, as can be seen from the survey paper |6|. Sensor location for state estimation was studied for example in |10|, using the trace norm of the error covariance of the optimum filter. The sensor location problem for identification of unknown parameters was solved in |9| with the aid of the determinant of the Fisher information matrix, and with application to a bubble-column loop bioreactor. Many other studies concerning these two types of problems are reported in |6|, together with the solutions given to the problem of optimum controller and sensor location for optimum control of a distributed parameter system.

1.3 Optimal sensor location for parameter identification

Up to our knowledge, the only study made from that point of view is reported in |13|, and is concerned with structural identification for both linear and non-linear systems. The proposed solution is based upon an optimization of the trace norm of the Fisher information matrix. It is important to note that, in this case, the measurement matrix H is a **selection** matrix (with coefficients equal to 0 or 1), and that the optimization is done by exhaustive search.

1.4 Optimal sensor location for failure detection

This problem is investigated in |15| in the framework of non-linear systems. A reduced order time-varying linear observer is designed for full state estimation in such a system. Inspection of the state estimates and/or several observer residuals leads to detection and diagnosis of the faults, without any statistical test. The optimum sensors location problem is then solved by exhaustive search for minimizing the observation cost associated to each set of measurements which is convenient for this fault detection strategy.

The dual problem of optimum actuator location in large space structures is considered in |14|. A degree of controllability, which accounts for possible component failures, is defined and optimized over the admissible set of controller location, either by exhaustive search or be solving an integer programming problem.

2. OUR APPROACH

In this paper, we address problem 1.4., namely the problem of optimal sensor location for detecting changes in the eigenstructure of a dynamical system. Because of our detection and identification approaches, we actually also address problem 1.3. The underlying application is vibration monitoring for offshore platforms; the interested reader is referred to |3| for a presentation of this application. We derived statistical instrumental tests for detection and diagnosis of changes in the vibrating characteristics of a structure subject to an unknown nonstationary excitation. The numerical performances of these tests are reported in |2| for scalar signals and |3| for multivariable systems. The theoretical properties of these tests are investigated in |12| under stationarity assumptions: the criterion which is used for evaluating the performances of the tests is the classical detection power for a fixed level (false alarm rate).

The purpose of this paper is the investigation of the possible uses of such a type of criterion for designing optimal numbers and locations of sensors. We especially emphasize the key points of choice of parametrization for optimization in section II, comparison of designs with different number of sensors in section III, and influence of the geometry of the excitation upon the optimal design in section IV. Numerical results obtained on a simulated structure are also reported in section IV. Conclusions are given in section V.

II. PROBLEM STATEMENT CHOICE OF PARAMETRIZATION

We consider a dynamical system described by the following discrete time state space representation:

$$\begin{cases} X_{t+1} = F\,X_t + V_{t+1} \\ Y_t = H\,X_t \end{cases} \tag{1}$$

where the state X is of dimension n, the observation Y is of dimension r << n, and where V_t is a gaussian white noise with covariance matrix Q. The observation matrix H is a selection matrix, i.e. we observe a limited number of state variables. The change or failure detection problem we solved in |3| and |12| is as follows: given a measurement matrix H, detect and diagnose changes in the state transition matrix F, or equivalently in the eigenstructure of the system, without knowing or using any estimate of the noise covariance matrix Q (which is furthermore time varying in |3|).

1. NEW TESTS FOR CHANGE DETECTION AND DIAGNOSIS

For this purpose, we derived new statistical tests which may be summarized in the following way. The multivariable process (1) may be equivalently represented by the ARMA model:

$$Y_t = \sum_{i=1}^{p} A_i\,Y_{t-i} + \sum_{j=0}^{p-1} B_j\,E_{t-j} \tag{2}$$

where (E_t) is a standard white noise. One possible way is to solve the following linear system of equations:

$$\sum_{i=0}^{p} A_i\,H\,F^{p-i} = 0 \tag{3}$$

with $A_0 = -I_r$.

In such a case, the change detection problem is to detect changes in the AR parameters A_i of (2), while considering the MA parameters B_j as nuisance parameters; furthermore, deciding which poles and corresponding eigenvectors have changed would solve the diagnosis problem. We use a model validation approach. Given a nominal AR model $\theta°$, where:

$$\theta^T = (A_p,\dots,A_1) \tag{4}$$

or a nominal observable model (H_0,F_0), and a sample of observations $Y_1,\dots,Y_S$, we consider what we call the **instrumental statistics**:

$$U_N(s) = \frac{1}{\sqrt{s}} \sum_{t=1}^{s} Z_t^N\,W_t^T \tag{5}$$

where: $Z_t^{NT} = (Y^T_{t-p},\dots,Y^T_{t-p-N+1})$

is the vector of instruments

$$W_t = Y_t - \theta^{oT}\,\phi_t$$

$$\phi_t^T = (Y^T_{t-p},\dots,Y^T_{t-1}).$$

We also introduce the corresponding vectors:

$$\Theta \overset{\Delta}{=} \mathrm{col}(\theta^T)$$

$$\mathcal{U}_N(s) \overset{\Delta}{=} \mathrm{col}\,(U_N(s)) = \frac{1}{\sqrt{s}} \sum_{t=1}^{s} Z_t^N \otimes W_t \tag{6}$$

1.1 Detection

The two hypotheses to be tested are :

H_0 : $\theta = \theta°$ no change

H_1 : $\theta = \theta + \frac{\delta\theta}{\sqrt{s}}$ small change in direction $\delta\theta$, ie local alternative .

It may be shown |3| that, under H_0, $U_N(s)$ (5) is zero-mean, and that, under H_1, we have:

$$E_1\,(U_N(s)) \simeq \mathcal{H}^T_{p,N}\;\delta\theta \tag{7}$$

where $\mathcal{H}_{p,N}$ is the Hankel matrix of the process (2) under H_0.

It may be shown that $U_N(s)$ allows the detection of any change in the minimal representation of (1) |12| |3|.

Furthermore, we also use the following **local** asymptotic normalities:

under H_0: $\mathcal{U}_N(s) \xrightarrow[s\to\infty]{\mathcal{L}} \mathcal{N}(0,\Sigma_N)$

under H_1: $\mathcal{U}_N(s) \xrightarrow[s\to\infty]{\mathcal{L}} \mathcal{N}((\mathcal{H}^T_{p,N} \otimes I_r)\;\delta\Theta\,,\;\Sigma_N)$

where

$$\Sigma_N = \sum_{1-p}^{p-1} E_0(Z_t^N\;Z_{t-i}^{N\,T} \otimes W_t\;W_{t-i}^T)$$

is the covariance matrix of $\mathcal{U}_N$.

We have thus transformed the initial change detection problem into a problem of detection of a change in the mean of a gaussian process. As discussed in |12|, the convenient test is then:

$$t_0(s) = \mathcal{U}_N^T(s)\;\Sigma_N^{-1}\,M(M^T\,\Sigma_N^{-1}\,M)^{-1}\,M^T\,\Sigma_N^{-1}\mathcal{U}_N(s) \tag{8}$$

where $M = \mathcal{H}^T_{p,N}\,D^T \otimes I_r$ (9)

and D is such that M is of full column rank nr. We refer the interested reader to |3| for details concerning the implementation of the global test (8) in the (real) situation where the noise covariance matrix Q in (1) -or equivalently the MA parameters B_j in (2)- are timevarying.

1.2 Diagnosis

For solving the diagnosis problem, namely deciding which eigenvalues and eigenvectors -of F in (1) or equivalently of the AR part in (2)- have changed, our approach is the following. We still use the instrumental statistics $\mathcal{U}_N(6)$ (5), together with a relationship between changes in the eigencharacteristics of the system and changes in the AR parameters Θ . As we look for small changes, we use first order Taylor expansions for Θ . Let ϕ be a minimal parametrization of the AR part of the process, for example the modal characteristics, namely the eigenvalues μ of F and the observed par $H\varphi$ of the corresponding eigenvectors. Assume that $\phi \to \Theta = f(\phi)$ is continuously differentiable in the neighborhood of the nominal model ϕ_0. For monitoring a particular subset of the coordinates of ϕ, we use the test given by formula (8) with:

$$M = (\mathcal{C}^T_{p,N}\, D^T \otimes I_r)\, \mathcal{J} \qquad (10)$$

where $\mathcal{J}$ is the matrix obtained by selecting the convenient columns of the jacobian matrix $f'(\phi_0)$. For example, it may be shown |3| |11| that writing equation (3) in the modal basis and differentiating it lead to the $\mathcal{J}$ of interest.

Examples of such $\mathcal{J}$ may be found in |3|. We call such type of tests "sensitivity tests". They will be of key importance for the sensor location problem, as will be seen in the next paragraph.

This approach for detection and diagnosis turns out to be very powerful, even for small changes in the eigencharacteristics. Numerical results may be found in |2| |3|. A detailed theoretical analysis of the performances of these tests may be found in |12|.

2. THE CRITERION TO BE OPTIMIZED

We now discuss the problem of optimal sensor location: given a reference model F_0 in (1), how to choose the best measurement (selection) matrix H in order to maximize the detection performances of the global test t_0 (8) (9) and/or the sensitivity tests (8) (10). As for investigating the theoretical properties of these tests T, the criterion we consider is the power β, for a fixed false alarm rate α. More precisely, we maximize:

$$\beta = \mathbb{P}_1 \ (T > \lambda) \qquad (11)$$

where the threshold λ is chosen according to:

$$\mathbb{P}_0 \ (T > \lambda) \leqslant \alpha \qquad (12)$$

Let us now emphasize that all the tests we have introduced have the following form. Let U be a gaussian random variable:

$$U \sim \mathcal{N}(\mu, \Sigma) \quad .$$

Assume first that:

$$\mu = M\nu \qquad (13)$$

is one parametrization of the mean value of U such that M is of full column rank m and $\nu \in R^m$.
The test of $H_0: \mu = 0$ against $H_1: \mu = M\nu$ is defined by:

$$T = U^T \Sigma^{-1} M(M^T\Sigma^{-1} M)^{-1} M^T \Sigma^{-1} U \qquad (14)$$

and is asymptotically distributed as a χ^2 variable with m degrees of freedom under both H_0 and H_1. Under H_0 T is centered, by under H_1 the distribution of T has a non centrality parameter equal to:

$$\gamma^2 = \nu^T \, \Gamma \, \nu \qquad (15)$$

where $\Gamma = M^T \Sigma^{-1} M$. (15')

If we now assume that the rank of M in (13) is q<m, let D be the matrix containing the basis vectors of a complement of the kernel of M. Then it may be shown |11| that, whatever the rank of M in (13) is, the non centrallity parameter is defined by (15).

Consequently, the threshold λ depends only upon the number of degrees of freedom (which is equal to nr for the global test (8) (9)), while the power β is an increasing function of γ (15), for a given λ . We will thus concentrate our efforts on the **optimization, with respect to the measurement matrix H, of the quadratic form defined by Γ (15')**.

2.1 Choice of parametrization

In order to optimize this quadratic form, we have to choose a scalar criterion. As many other authors (see Section I), we have chosen the trace norm and thus the following criterion:

$$C(M) = \mathrm{Tr}(\Gamma) = \mathrm{Tr}(M^T \Sigma^{-1} M) \qquad (16)$$

This choice may be justified using the following remark |8|:

$$\int \nu^T M^T \Sigma^{-1} M\nu \, d\nu = C(M) \ . \ C_m/m \qquad (17)$$

where we integrate over the unit sphere in R^m and where C_m is the area of this sphere. In other words, the criterion C(M) is nothing but the mean value of the non-centrality parameter when ν covers the unit sphere in R^m.

The key point here is that the criterion C(M) does depend upon the parametrization which is chosen for the mean value μ of the instrumental statistics $\mathcal{U}$ under H_1, since C(MP) is generally different from C(M) even if P is a change of basis. The first basic consequence, as far as optimal sensor location is concerned, is that it is necessary to choose a **parametrization** of the mean of $\mathcal{U}$ **which does not depend upon the location** of the sensors. From (3) we can see that the ARMA representation (2) depends upon the sensors location. Thus we conclude that the criterion:

$$C_N = \mathrm{tr}(\Gamma_N) = \mathrm{Tr}\left[(\mathcal{H}_{p,N} \otimes I_r)\ \Sigma_N^{-1}\ (\mathcal{H}^T_{p,N} \otimes I_r)\right]$$

which was used in |12| for optimization over N for fixed H, is no more convenient for optimizing over H.

From now on, we will thus consider parametrizations related to the own characteristics of the system: eigen (or modal) parameters, state transition matrix F. Before doing that, we notice that the following invariance property of the criterion C (16):

$$P\,P^T = I_m \Rightarrow C(MP) = C(M) \quad . \qquad (18)$$

2.2 Several possible criterions

According to the previous discussions about the choice of parametrization for sensors location in paragraph 2.1 and, on the other hand, about diagnosis via sensitivity tests in paragraph 1.2., it results that several criterions of the type (16) may be considered, using various jacobian matrices $\mathcal{J}$ in M (10).

2.2.1 Global modal sensitivity

One subset of parameters which is of particular interest, as far as vibration monitoring is concerned, is the set of the modal parameters, namely the vibrating pulsations ω_j and the eigenvectors ψ_j. Of course, only the observed part $H\ \psi_j$ of the eigenvectors can be monitored, but we nevertheless choose the whole set (ω_j, ψ_j) to parametrize the system independently of the observation matrix H. We have chosen not to monitor the damping coefficients c_j (real part of the eigenvalue λ_j) because they are usually not precisely identified. In this case, the mean value of $\mathcal{U}_N$ under H_1 has the following parametrization |11| |12|

$$\mathbb{E}_1(\mathcal{U}_N) = (\mathcal{C}_N^{*T} \otimes I_r)\,\mathcal{J}\,\mathcal{L}\begin{pmatrix} d\psi re \\ d\psi im \\ d\omega \end{pmatrix}$$

where $\mathcal{C}_N^*$ is the controllability matrix of (1) in the modal basis, and $\mathcal{L}$ is such that $\mathcal{L}\mathcal{L}^T = I$.

Thus, because of the discussion following formulas (15) and of (18), the global modal criterion is defined according to (16) as:

$$C_{\omega,\psi} = \mathrm{Tr}\left[\mathcal{J}^T\ (\mathcal{C}_N^* \otimes I_r)\ \Sigma_N^{-1}\ (\mathcal{C}_N^{*T} \otimes I_r)\mathcal{J}\right] . \qquad (19)$$

It is of interest to notice that, because the block decomposition of $\mathcal{J}$ involves block diagonal matrices |11|, we have:

$$C_{\omega,\psi} = \sum_{j=1} C_j \qquad (19')$$

where C_j is the sensitivity criterion corresponding to only one mode, i.e. one pulsation and one eigenvector |11|. Numerical results concerning the criterion (19) will be presented in Section IV. The optimization is done by exhaustive search.

2.2.2 Sensitivity w.r.t. the F matrix

Using the same approach as in the previous paragraph, we now differentiate (3) to obtain a connection between variations in $\textcircled{H}$ and variations in F through a Jacobian matrix J_F.

The sensitivity criterion with respect to the variations in F is thus:

$$C_F = \mathrm{Tr}\left[J_F^T \; (\mathcal{C}_N \otimes I_r) \, \Sigma_N^{-1} \; (\mathcal{C}_N^T \otimes I_r) \; J_F \right].$$

As $\mathcal{Y}$ in (19) and J_F have respectively (2r + 1) l and $(2l)^2$ columns (n=2l), and because r<<l, the criterion C_F is much more computationaly expensive than the criterion $C_{\omega,\psi}$ (19).

III. COMPARISONS OF DESIGNS WITH DIFFERENT NUMBER OF SENSORS

According to the discussion presented in Section II.2, up to now we have basically been able to compare different sensor locations corresponding to the **same** number of sensors: recall that the criterion (16) is directly related to the test power β (11), which is defined for a fixed threshold λ, and that λ depends upon the number q of degrees of freedom.

The purpose of this section is to define a correcting factor with allows the comparison between different number of sensors.

First notice that the power β (11) is computed as:

$$\beta = P(x_1^2 + \ldots + x_q^2 \geqslant \lambda) \triangleq P(\mathcal{D}_\lambda)$$

where $x_i = z_i + \gamma_i$

and (z_i) are zero mean independent identically distributed gaussian variables, and γ_i is the mean of x_i.

Because of symmetry, we have:

$$P(\mathcal{D}_\lambda) = P(x_1^2 + \ldots + x_q^2 \geqslant \lambda) = P((z_1+\gamma)^2 + z_2^2 + \ldots + z_q^2 \geqslant \lambda) \quad (20)$$

where $\gamma^2 = \gamma_1^2 + \ldots + \gamma_q^2$ is the noncentrality parameter.

Since we consider only small changes, i.e. γ is small, we can keep only the first two nonzero terms of the Taylor expansion of (20): $\beta = \alpha + \gamma^2 \, \varepsilon_q/2 + 0(\gamma^3)$, where α is the level (12).

It may be shown |11| that:

$$\varepsilon_q = \exp(-\delta^2/2) \; / \; \sqrt{2\pi q}$$

where $\delta = \phi^{-1}(1-\alpha)$ and ϕ is the gaussian cumulative distribution function.

Since δ does not depend upon q, the convenient quantity to compare locations of different number of sensors is: $(\beta - \alpha)\exp(\delta^2/2)$ which is equal to: $\gamma^2/2\sqrt{2\pi q}$ up to second order.

Because of (15) and (17), integrating the two sides of this last equality leads to:

$$(\beta-\alpha)\exp(\delta^2/2) = C(M) \; / \; 2\sqrt{2\pi q} \quad . \quad (21)$$

Therefore, **the criterion (16) has to be divided by the square root of the number of degrees of freedom.**

Numerical results involving this criterion will be presented in the next section.

IV. NUMERICAL RESULTS

In this section, we present numerical results concerning the criterion (19) computed for a simplified platform model. This simulation model is a nonsymmetric tied down system of 18 masses of one degree of freedom, connected by springs, as shown in figure 1, with known weights, stiffness and damping coefficients. No signals were generated; the theoretical values of this criterion were computed from the physical characteristics of the system and for a given excitation. Actually, it may be shown that everything in (19) can be computed as functions of (H,F) and the theoretical covariances R_k of the observation Y. We insist upon the fact that, even though the tests (8) may be computed without knowing the excitation V in (1), our criterion does depend upon the excitation (through its covariance Q). This dependency is analytically complex, and thus will be studied only via numerical computations made with four different covariance matrices Q.

In order to mimic the effect of the swell, and assuming that the excitations at different points are independent, we selected diagonal covariance matrices $Q_i (1 \leqslant i \leqslant 4)$. Excitation 1 is stronger than excitation 2 on the top level of the structure. Excitation 3 trys to simulate a dominant excitation on the "leg" 3-9-15. Excitation 4 acts in a similar way on the leg 6-12-18.

As we have previously mentioned, the optimization has been done by exhaustive search among a set of possible sensor locations. According to experiments currently performed on real offshore platforms, the locations which we have used correspond to the selection of r=2, 3 or 4, a total number of sensors located on each of the two opposite "legs" 1-7-13 and 6-12-18, with 1 or 2 sensors on each leg, resulting in a set of 36 possible locations.

The four global modal criterions $C_i (1 \leqslant i \leqslant 4)$ have been computed applying formula (19'), and then multiplied by the correcting factor $1/\sqrt{m}$ according to (21). Here m=36r-18, because we have chosen p to satisfy: rp=n=36 and N=p and because we have omitted the damping coefficients c_j. The values of these four global criterions are plotted on figure 2.

We first notice that the four curves of figure 2, are quite similar, as far as their maxima are concerned, showing a not too strong dependency of the criterions upon the excitation. Another way of checking this relative independency consists in sorting, for each excitation, the different locations according to the decreasing values of the global modal criterion. If the criterion $C_{\omega,\psi}$ was independent upon the excitation, we would get the same rank value $\mathcal{R}_i$ on each row (i.e. for each location). We have shown |11| that the real situation is not dramatically different from this ideal situation: see table 1.

Two other important facts may also be deduced from table 1. First, it does not seem really necessary to have 4 sensors for good detection, because good scores are obtained with 3 sensors coveniently located. Second, sensors locations which do not involve mass number 1 always get bad scores.

CONCLUSION

We have addressed the problem of optimal sensor location from the nonstandard point of view of failure detection with statistical tests. We have derived criterions based upon the power of the detection and diagnosis tests we recently developed for vibration monitoring |3| |11|. Several key points have been discussed and solved, namely: choice of parametrization for optimization of sensor location, comparison of designs with different number of sensors, influence of the excitation.

Further investigations should include deeper understanding of the adequacy of the mean criterion (17): actually we compute the mean power of our test for detecting any type of change with unit "magnitude". Our opinion is that a more convenient criterion could be obtained using Jacobians with respect to physical parameters (masses and stiffness parameters). This point is currently under investigation.

Finally, it is of interest to notice that, because of our approach for change detection and diagnosis, we have also addressed in this paper the problem of optimal sensor location for parameter identification. Actually, we show in |12| that the inverse of the matrix Γ in (15), which characterizes the asymptotic power of the instrumental test (14), is equal to the asymptotic covariance matrix of the estimation error of the optimal instrumental variable identification method. As we have addressed the optimal sensor location problem using as a criterion the power of the instrumental test, we also solved the problem of optimal sensor location for parameter identification.

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locations	$\mathcal{R}_1$	$\mathcal{R}_2$	$\mathcal{R}_3$	$\mathcal{R}_4$
1. 6. 0. 0.	10.	12.	11.	12.
1.12. 0. 0.	15.	17.	17.	16.
1.18. 0. 0.	23.	22.	24.	23.
7. 6. 0. 0.	24.	23.	22.	22.
7.12. 0. 0.	36.	35.	36.	36.
7.18. 0. 0.	33.	32.	33.	33.
13. 6. 0. 0.	25.	25.	25.	25.
13.12. 0. 0.	35.	33.	35.	34.
13.18. 0. 0.	34.	34.	34.	35.
7.13. 6. 0.	18.	19.	19.	18.
7.13.12. 0.	32.	31.	32.	31.
7.13.18. 0.	27.	28.	29.	28.
1.13. 6. 0.	12.	11.	12.	10.
1.13.12. 0.	8.	8.	8.	6.
1.13.18. 0.	21.	21.	21.	20.
1. 7. 6. 0.	4.	5.	5.	5.
1. 7.12. 0.	3.	4.	1.	1.
1. 7.18. 0.	16.	16.	16.	13.
1.12.18. 0.	7.	9.	9.	8.
1. 6.18. 0.	11.	10.	13.	11.
1. 6.12. 0.	1.	2.	3.	4.
7.12.18. 0.	31.	30.	31.	32.
7. 6.18. 0.	19.	18.	18.	19.
7. 6.12. 0.	17.	13.	14.	14.
13.12.18. 0.	28.	29.	28.	29.
13. 6.18. 0.	22.	24.	23.	24.
13. 6.12. 0.	20.	20.	20.	21.
7.13.12.18.	30.	36.	30.	30.
7.13. 6.18.	26.	27.	27.	27.
7.13. 6.12.	29.	26.	26.	26.
1.13.12.18.	13.	14.	10.	9.
1.13. 6.18.	14.	15.	15.	15.
1.13. 6.12.	2.	3.	2.	3.
1. 7.12.18.	6.	7.	7.	17.
1. 7. 6.18.	5.	6.	6.	7.
1. 7. 6.12.	9.	1.	4.	2.

Table 1:

Sensors locations and rank of these locations.

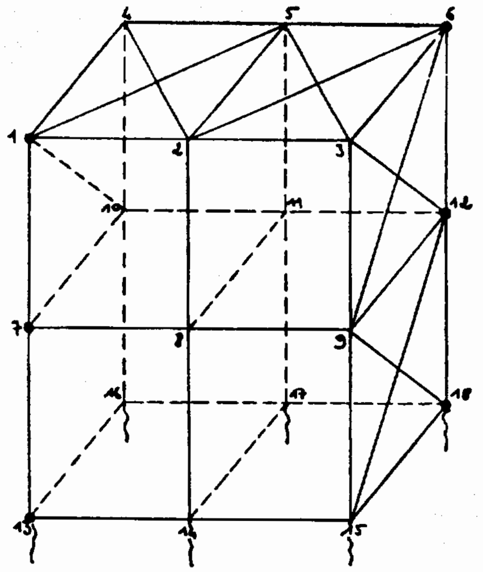

Figure 1 : Simulation system.

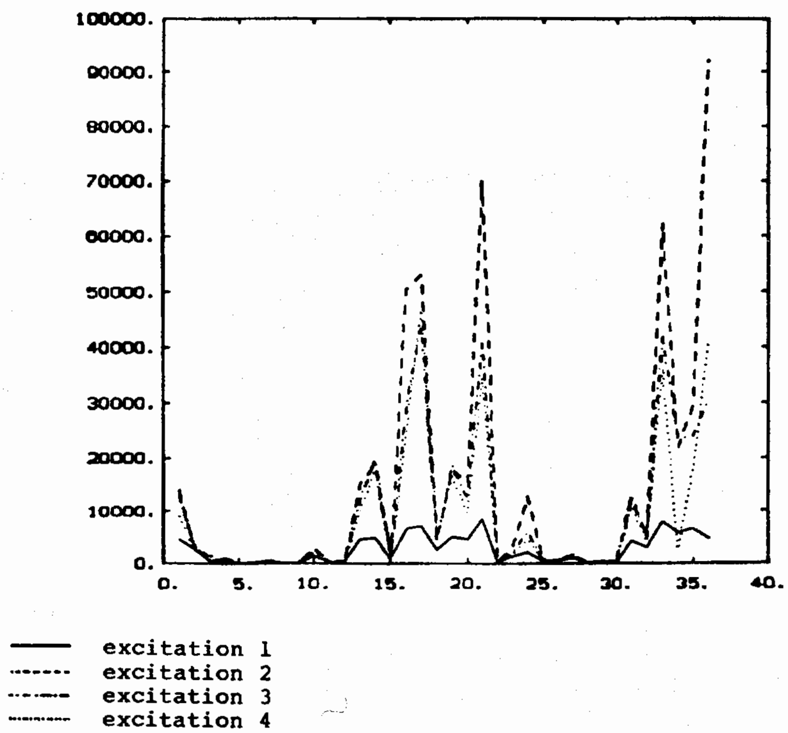

Figure 2 : The four global modal criterions.