ROBUST DETECTION OF KNOWN SIGNALS IN ASYMMETRIC NOISE

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ABSTRACT

The detection of signals in noise with possibly asymmetric probability density functions is considered. The noise density model allows a symmetric contaminated-nominal central part and an arbitrary tail behavior. For detection of known signals the robust likelihood-ratio (LR)-type detector is obtained, based on detector efficacy as performance criterion. The robust M-detector structure for constant-signal detection is also explicity obtained.

I. INTRODUCTION

Following the fundamental works of Huber on robust estimation [1] and robust hypothesis testing [2], many further developments and applications of robustness theory have been formulated by researchers in the communications sciences. Concepts of robustness in signal processing applications were certainly in existence prior to Huber's results [e.g., 3, 4]. However, it is generally accepted that the techniques and results in [1, 2] formed an important basis for much of the considerable subsequent research activity on robust schemes for signal estimation, detection and filtering applications. A recent survey paper [5] lists a large number of references on robust techniques.

In [6] Huber's ideas were applied to obtain the structures of asymptotically robust signal detectors. This resulted specifically in the canonical limiter-correlator detector for a weak deterministic signal in nominally Gaussian noise modeled as having a mixture or contaminated probability density. In [7] this result was extended to apply to other nominal noise densities. Both [6] and [7] considered detection structures of the type where the sum of memoryless transformations of each discrete-time input observation (the test statistic) is compared to a fixed threshold. For example, the limiter-correlator robust detector for a signal vector $(s_1, s_2, \ldots s_n)$ in an observation vector $(X_1, X_2, \ldots X_n)$ with independent and identically distributed additive noise computes the test statistic $T_n = \sum_{i=1}^{n} L_i(X_i)$, where $L_i(X_i) = s_i \ell(X_i)$ and ℓ is a limiter characteristic. We will call such detector structures LR-detectors, since this is the usual structure for a likelihood-ratio test on independent observations. More recently Huber's results were used in [8, 9] to obtain directly the robust M-detectors for both the fixed-sample and sequential cases. An M-detector structure is obtained when the detection test statistic Q_n is obtained as that function of the observations minimizing $\sum_{i=1}^{n} M(X_i - s_i Q_n)$, where M is some appropriately chosen function. Note that Q_n may be used as an estimator for the signal amplitude, and such an estimator is called an M-estimator because of its similarity to maximum likelihood estimators in general.

Two major factors limit the applicability of such results for signal processing schemes. One of these is the requirement of independence for the sequence of discrete-time input data to the detectors. This requirement of independence has recently been addressed in [10], where it was shown that robust detector structures can be derived for operation under conditions of

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weak dependence in the input sequence. The results in [10] were developed for detection applications following similar considerations which had earlier been applied in [11, 12] for robust estimation. The second main limitation of many previous results on robust detection has been the assumption that the allowable noise density functions are symmetric. In this paper we will be concerned with this latter problem, and we will develop the structures of the robust LR- and M-detectors for robust detection of weak deterministic signals with a noise model allowing asymmetry in the univariate noise density functions. Our study was largely motivated by some recent work on robust estimation with asymmetrically distributed noise [13, 14]; in particular, we will adapt and draw upon the techniques and results in [14] for this work.

Specifically, we consider the class $\hat{F}_{g,\varepsilon,d}$ of noise densities f given by

$$\hat{F}_{g,\varepsilon,d} = \{ f \mid f = \begin{cases} (1-\varepsilon)g + \varepsilon h, \text{ on } [-d,d] \\ \text{arbitrary, outside } [-d,d] \end{cases}, h \in \hat{H} \} \tag{1}$$

Here $\varepsilon \in (0,1)$ is the maximum degree of contamination of a nominal density function g. The density g is assumed to be strongly unimodel (i.e., $-\log g$ is convex) and symmetric, and in addition we assume it to be sufficiently regular so that g' is absolutely continuous. The parameter d is a positive parameter specifying the interval around the origin in which the noise density f is a bounded, symmetric, contaminated version of g. The class $\hat{H}$ is the class of all bounded, symmetric, density functions. Note that a valid $f \in \hat{F}_{g,\varepsilon,d}$ could be zero on $(-\infty,-d)$, and place a probability of $2(1-\varepsilon)[1-G(d)] + \varepsilon$ on (d,∞), where G is the distribution function corresponding to g. If g is the zero-mean Gaussian density with variance σ^2, a reasonable specification of d may be a number between 2σ and 4σ, and ε is typically between 0.001 and 0.1.

In the next sections we will consider the robust LR- and M-detectors for the noise model of (1).

II. ROBUST LR-DETECTOR FOR ASYMMETRIC NOISE DENSITIES

For a vector of observations $(X_i, X_2,\ldots,X_n)$ of length n, described by

$$X_i = N_i + \theta s_i \quad , \qquad i = 1,2,\ldots,n$$

where $(s_i,s_2,\ldots,s_n)$ is a deterministic signal vector and the N_i are independent and identically distributed noise components, we want to test the null hypothesis H_0 that $\theta=0$ versus the alternative H_1 that $\theta > 0$. For an LR-detector using test statistic

$$T_n = \sum_{i=1}^{n} s_i \ell(X_i),$$

we want to obtain the characteristic ℓ which results in a robust detector, for allowable univariate noise density functions f in the class $\hat{F}_{g,\varepsilon,d}$. As a criterion of performance we will use the detector efficacy $E(f,\ell)$ which is dependent on f and ℓ, and defined as [15]

$$E(f,\ell) = \lim_{n\to\infty} \frac{[\frac{d}{d\theta}E_\theta\{T_n\}|_{\theta=0}]^2}{\mathrm{var}_\theta\{T_n\}|_{\theta=0}} \tag{2}$$

It is clear that for an LR-detector to be consistent for all $f \in \hat{F}_{g,\varepsilon,d}$, the characteristic ℓ has to vanish outside $[-d,d]$. Since $\hat{F}_{g,\varepsilon,d}$ consists of densities symmetric on $[-d,d]$, we additionally require that the allowable ℓ are symmetric. Let L_c denote the class of all LR-detector characteristics ℓ satisfying

i) $\ell(x) = 0, \ |x| \geq c$,

ii) $\ell(x) = -\ell(-x)$,

iii) ℓ is absolutely continuous,

the parameter c being a non-zero cut-off value, $c \leq d$; the value of c is set by consistency requirements, as we will discuss soon.

Solution for Efficacy-Robust Detector

We want to find a least-favorable density f_R in $\hat{F}_{g,\varepsilon,d}$ and a corresponding optimum characteristic ℓ_R in L_c such that

$$\inf_{f \in \hat{F}_{g,\varepsilon,d}} E(f,\ell_R) = E(f_R,\ell_R) \tag{3}$$

Note that we will then have

$$E(f_R,\ell_R) = \sup_{\ell \in L_c} E(f_R,\ell) \tag{4}$$

since we require ℓ_R to be optimum for f_R.

The following theorem establishes the condition under which a pair (f_R,ℓ_R) can be found in $\hat{F}_{g,\varepsilon,d} \times L_c$ satisfying (3) and (4) with a finite, non-zero value $E(f_R,\ell_R)$. It is directly related to Theorem 3.1 in [14].

Theorem 1: If the condition

$$\varepsilon < (1-\varepsilon)\{2cg(0) - [2G(c)-1]\} \tag{5}$$

is satisfied, where G is the distribution function corresponding to g, the density function

$$f_R(x) = \begin{cases} (1-\varepsilon)g(x) & , |x| \leq a_0 \\ \dfrac{(1-\varepsilon)g(a_0)}{\cosh^2[\frac{1}{2}a_1(c-a_0)]} \cosh^2[\frac{1}{2}a_1(c-|x|)], & a_0 < |x| \leq c \\ \text{arbitrary} & , |x| > c \end{cases} \tag{6}$$

and the corresponding optimum characteristic in L_c

$$\ell_R(x) = \begin{cases} \dfrac{-f_R'(x)}{f_R(x)} & , |x| \leq c \\ 0 & , |x| > c \end{cases} \tag{7}$$

satisfy (3) and (4), with $0 < a_0 < c$ and $\dfrac{-g'(a_0)}{g(a_0)} < a_1$ being the unique solutions of

$$\varepsilon = \int_{-c}^{c} f_R(x)dx - (1-\varepsilon)\int_{-c}^{c} g(x)dx$$

and

$$\frac{-g'(a_0)}{a_1 g(a_0)} = \tanh[\tfrac{1}{2}a_1(c-a_0)]$$

Comments on Theorem 1. If condition (5) is not satisfied, then f_R can be picked to be a constant on $[-c,c]$ so that $E(f_R,\ell)=0$. In [14, Table 1] and in Table 1, some numerical values are given for the upper bounds $\varepsilon(c)$ on ε, for different c, for which (5) is satisfied with g the unit variance Gaussian density. The value of $\varepsilon(c)$ is larger than 0.1 if c is larger than unity. The proof of Theorem 1 is a direct extension of the proof of Theorem 3.1 in [14], which placed more restrictive conditions on the allowable density functions and estimator characteristics. Our criterion of detection efficacy is used as an estimator criterion in [14]; however, its interpretation as an estimation variance for the statistics in [14] requires further assumptions which are not simply characterized as conditions on the allowable density functions. We will elaborate on this in the next section.

For $c \leq d$, it can be easily shown that ℓ_R of (7) results in a test which is consistent for θ in a positive neighbourhood of the origin. To see this, it is sufficient to show that the slope at $\theta=0$ of the mean function of ℓ_R, given by

$$m'(0) = \int_{-c}^{c} \ell_R'(x) f(x)dx,$$

is positive for all $f \in \hat{F}_{g,\varepsilon,d}$. Now under the condition of Theorem 1, $E(f_R, \ell_R)$ is positive, and $\int_{-c}^{c} \ell_R'(x) f_R(x)dx$ is positive. Since $\hat{F}_{g,\varepsilon,d}$ is convex, it follows that $m'(0) > 0$ for all $f \in \hat{F}_{g,\varepsilon,d}$.

Now even if $m'(0)$ is positive, the mean function $m(\theta) = \int_{-c}^{c} \ell_R(x) f(x-\theta)dx$ will become zero for some positive value θ_{max} of θ. In Table II we show the computed value of θ_{max} as a function of c and ε, for the case $c=d$, with g again the unit-variance Gaussian density. These values were obtained by considering for each θ the noise density minimizing $m(\theta)$. These results indicate that for given ε, increasing d with $c=d$ leads to a test consistent for an increasing range of values of θ; in the limit, we get consistency for all θ and the solution degenerates to that in [6,7].

It has not been possible to prove a stronger result which simultaneously bounds the worst-case asymptotic local slope of the power function and the false-alarm probability as in [6,7], where the fixed threshold of the robust LR-detector could also be determined. The reason why the stronger result is not possible here is that ℓ_R is not a monotone limiter characteristic, so that the numerator and denominator in (2) are not separately minimized and maximized, respectively, by f_R when $\ell = \ell_R$. In the next section we consider an M-detector robust structure which allows a stronger robustness property to be derived.

III. ROBUST M-DETECTOR FOR ASYMMETRIC NOISE DENSITIES.

We will restrict attention to the special case of constant signals, and without further loss of generality we will take $s_i=1$, $i=1,2,\ldots n$. The general case of non-constant signals requires further considerations, as we will indicate later. Our M-detectors will therefore be based on statistics Q_n satisfying

$$\sum_{i=1}^{n} \psi(X_i - Q_n) = 0, \qquad (8)$$

so that we are implicitly assuming M to be sufficiently regular.

We are interested in the class $F_{g,\varepsilon,d}$ (or a useful subset of the class $\hat{F}_{g,\varepsilon,d}$) of densities, and we impose the following reasonable constraints in defining the class Ψ_c of allowable M-detector functions ψ we will consider:

(i) $\psi(x) = 0, \ |x| \geq c$,
(ii) $\psi(x) = -\psi(-x)$,
(iii) $\psi'(x)$ is bounded and piecewise-continuous on $[-c,c]$.

An additional constraint will be added soon. The value of the parameter $c < d$, which defines the size of Ψ_c, is set by requirements we consider next.

To complete the specification of our class of M-detectors a solution scheme has to be specified for obtaining Q_n satisfying (8). This is necessary because a soluiton to (8) may not be unique. The scheme we specify is a simpler iterative procedure than that considered in [14]. Although its numerical convergence rate to the solution may be somewhat smaller, it allows us to obtain more explicit robustness results than were obtained in [14] (specifically, note remark 3.5 in [14]).

We define the test-statistic Q_n of an M-detector based on $\psi \in \Psi_c$ in terms of the sequence $\{Q_n^j\}$ given by

$$Q_n^{j+1} = Q_n^j + \frac{\lambda_n(Q_n^j)}{D}, \quad j=0,1,2,\ldots, \tag{9}$$

where

$$\lambda_n(q) = \int_{-\infty}^{\infty} \psi(x-q)dF_n(x), \tag{10}$$

F_n being the empirical distribution function of the n observations, with D being a positive constant. The solution Q_n is defined as $Q_n = \lim_{j\to\infty} Q_n^j$, with Q_n^0 a sample median (or any consistent estimator of the median of the X_i), provided the limit exists. Otherwise Q_n is taken to be Q_n^0.

In terms of the quantities g, ε and d defining $\hat{F}_{g,\varepsilon,d}$ we now define a new parameter k_0 by

$$k_0 = G^{-1}\left[\frac{1}{2} + \frac{\tau}{2(1-\varepsilon)}\right] \tag{11}$$

with

$$\tau = 2(1-\varepsilon)\ [1-G(d)] + \varepsilon, \tag{12}$$

or directly as

$$k_0 = G^{-1}\left[1-G(d) + \frac{1}{2(1-\varepsilon)}\right] \tag{13}$$

G being the distribution function corresponding to the density g. Note that τ is the maximum value of the total probability which may be distributed, arbitrarily, outside [-d,d]. We will assume that our model is such as to make τ < 0.5, so that the median of any $f \in \hat{F}_{g,\varepsilon,d}$ lies in (-d,d). This implies the restriction ε < 0.5. From (12) we have

$$d = G^{-1}\left[\frac{1}{2} + \frac{1-\tau}{2(1-\varepsilon)}\right],$$

so that $k_0 < d$. Under this assumption (τ<0.5) the maximum value of $|m|$, where m is the median of $f \in \hat{F}_{g,\varepsilon,d}$, satisfies

$$[(1-\varepsilon)[G(|m|)-G(-d)] = \frac{1}{2}$$

so that

$$G(|m|) = 1-G(d) + \frac{1}{2(1-\varepsilon)},$$

and thus the median is always in $[-k_0,k_0]$, from (13).

Our objective now is to establish conditions for the asymptotic normality of Q_n based on $\psi \in \Psi_c$ for $f \in \hat{F}_{g,\varepsilon,d}$. We will finally be able to define subsets of Ψ_c and $\hat{F}^c_{g,\varepsilon,d}$ over which asymptotic normality holds, and we will obtain the saddlepoint solution for the asymptotic variance over these classes. The following results are stated without proofs; proofs will be given in a detailed version of the paper [16].

We consider first the consistency of Q_n.

Lemma 1. Let $c \leq d - k_0$ and ψ be any characteristic in Ψ_c. Suppose that for a given $f \in \hat{F}_{g,\varepsilon,d}$ the median lies in the open interval $(-k_0, k_0)$ and the function

$$\lambda(q) = \int_{-c+q}^{c+q} \psi(x-q) f(x) dx$$

is strictly decreasing on $[-k_0, k_0]$. Then for $D > \frac{1}{2} \max_{[-c,c]} |\psi'(x)|$ in (9), the M-detector based on ψ has for this f a test statistic Q_n which converges in probability to θ; in addition,

$$\lim_{n\to\infty} P\{\sum_{i=1}^{n} \psi(X_i - Q_n) = 0\} = 1$$

Note that this lemma implies that the iterations for Q_n in (9) will converge (and therefore Q_n is not defined as Q_n^0) with a probability approaching unity as $n \to \infty$.

The following lemma is concerned with asymptotic normality of Q_n.

Lemma 2. Let (ψ, f) be a pair in $\Psi_c \times \hat{F}_{g,\varepsilon,d}$, with $c \leq d - k_0$, for which the conditions of Lemma 1 are satisfied. If in addition we have $\lambda'(q) < 0$ in a closed neighbourhood of the origin, then $\sqrt{n}(Q_n - \theta)$ is asymptotically normally distributed with variance $V(f,\psi)$ given by

$$V(f,\psi) = \frac{1}{E(f,\psi)} = \frac{\int_{-c}^{c} \psi^2(x) f(x) dx}{\left[\int_{-c}^{c} \psi'(x) f(x) dx\right]^2}$$

Lemma 2 follows from results in [17, Section 4] where general conditions are given ensuring asymptotic normality of M-estimators. That these conditions are met under Lemma 2 can be easily demonstrated.

We are now ready to obtain a least-favorable density and corresponding optimum M-detector characteristic which together form a saddlepoint for performance in terms of asymptotic variance of a consistent and asymtotically normal detection test statistic. First we define $\psi_R \in \Psi_c$ by $\psi_R(x) = \ell_R(x)$ of (7), where $f_R(x)$ was defined in (6). Then we have

Lemma 3. Let g, ε, d be such that $\tau < \frac{1}{2}$ in (26) and k_0 of (13) is less than $\frac{a_0}{2}$, for $c \leq d - k_0$ satisfying (5). [The parameter a_0 is present in the definition of $\psi_R = \ell_R$ in (7).] Then for $f = f_R$ and $\psi = \psi_R$ the test statistic Q_n obtained from (9) with $D > \frac{1}{2} \max_{[-c,c]} |\psi_R'(x)|$ is consistent for θ, and $\sqrt{n}(Q_n - \theta)$ is asymptotically normally distributed with variance

$$V(f_R, \psi_R) = \frac{\int_{c}^{-c} \psi_R^2(x) f_R(x) dx}{\left[\int_{-c}^{c} \psi_R(x) f_R'(x) dx\right]^2} \tag{14}$$

Let $\mathcal{F}^*_{g,\varepsilon,d} \subset \hat{\mathcal{F}}_{g,\varepsilon,d}$ be the subset of densities in $\hat{\mathcal{F}}_{g,\varepsilon,d}$ which are strictly unimodal on $[-d,d]$. In addition, the parameters ε and d are restricted to satisfy the condition $\tau < 0.5$, with τ defined in (12), and the medians are assumed to lie in $(-k_0,k_0)$, with k_0 defined in (13). This last condition requires positive probabilities to be distributed both on $(-\infty,-d)$ and on (d,∞), and is an insignificant restriction. Finally, with $c = d-k_0$, the parameter a_0 defining f_R in (6) is assumed to be larger than $2k_0$ and also (5) is assumed to be satisfied. It is easy to show that these conditions are satisfied for reasonable choices of g, ε, and d. For example, let g be the zero-mean, unit-variance Gaussian density, let $\varepsilon = 0.05$ and $d = 2$. Then $\tau = 0.093$ and $k_0 = 0.123$. With $c = d-k_0 = 1.877$ (5) is satisfied, and the value of $\frac{a_0}{2}$ is 0.502.

We finally restrict consideration to the subset $\Psi^*_c \subset \Psi_c$, containing M-detector characteristics which are non-negative on $(0,c)$ in addition to satisfying the three conditions defining ψ_c. This is a reasonable restriction, in view of the strict unimodality restriction used in defining $\mathcal{F}^*_{g,\varepsilon,d}$.

With the specification of g, ε, and d implicitly defining $c = d-k_0$, we get the following result:

Theorem 2. (i) Let $\psi_R \in \Psi^*_c$ be defined by $\psi_R = \ell_R$ of (7), and let Q_n be the test statistic arising from the M-detector based on ψ_R, with $D > \frac{1}{2} \max_{[-c,c]} |\psi_R'(x)|$. Then for any $f \in \mathcal{F}^*_{g,\varepsilon,d}$, Q_n is a consistent estimator for θ; $\sqrt{n}(Q_n-\theta)$ is asymptotically normally distributed with variance $V(f,\psi_R)$ satisfying

$$\max_{f \in \mathcal{F}^*_{g,\varepsilon,d}} V(f,\psi_R) = V(f_R,\psi_R),$$

where $V(f_R,\psi_R)$ was defined in (14).

(ii) For any $\psi \in \Psi^*_c$ which gives a consistent statistic for all $f \in \mathcal{F}^*_{g,\varepsilon,d}$,

$$V(f_R,\psi_R) \leq V(f_R,\psi)$$

where $V(f_R,\psi)$ is the asymptotic variance of the normalized statistic based on ψ.

Theorem 2 provides the main result of this section. We started from classes $\hat{\mathcal{F}}_{g,\varepsilon,d}$ and Ψ_c of densities and detector characteristics, respectively, but our least-favorable density and robust characteristic were obtained from amongst functions in $\mathcal{F}^*_{d,\varepsilon,g}$ and Ψ^*_c, respectively. The essential restriction added was that the densities considered be strictly unimodal on $[-d,d]$; this is not unreasonable, because the nominal density g is assumed to have this property. In applying the robust statistic Q_n arising from ψ_R in signal detection, one has to set a threshold based on the maximum variance $V(f_R,\psi_R)$. Thus the false-alarm probability constraint is automatically satisfied. In addition, for any $f \in \mathcal{F}^*_{g,\varepsilon,d}$, the asymptotic power function or the slope of the power function at the origin can be lower bounded by the corresponding values for $f = f_R$, depending on specific conditions on the signal strength parameter and detection threshold values. The details can be found in [8]; the main condition has been proved in Theorem 2 (specifically, the condition in [8, Lemma 2]).

The major reason why we confined attention to the constant-signal case in this section is that we need a reasonable initial value (e.g., the median Q_n^0) in starting the iterations in (9), to guarantee a consistent statistic Q_n. Extension of our results to the general case seems possible, and would appear to require an initial estimate of the median based on some nonparametric or other simple regression procedure [18,19]. The consistency proofs and conditions would also have to be extended.

IV. CONCLUSION

We have derived the structures of robust LR- and M-detectors for known-signal detection in noise for which the probability density has a symmetric, contaminated central part and arbitrary tail behavior. This model has been used previously for robust estimation studies.

The robust LR-detector was derived for performance defined by detection efficacy, a weak-signal large sample-size asymptotic performance measure. Although the detection efficacy is directly related to the slope of the detector power function, it was not possible to obtain simultaneous control on the false-alarm probability.

For constant-signal detection, the robust M-detector characteristic was obtained for performance characterized by the asymptotic variance of the test statistic. This result for the asymptotic variance, together with earlier results on M-detectors, allow more interesting robust detection solutions which can maintain the false-alarm probability within desired upper bounds. Our robustness results differ from previous results on robust estimation which also examined asymptotic variance in that we considered a simpler solution strategy and obtained the explicit saddlepoint solution for well-defined classes of noise and detector characteristic.

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c	ε (c)
1.0	0.103
1.5	0.248
2.0	0.391
2.5	0.502
3.0	0.583

TABLE I. Upper Bound on ε as Function of c for which (5') holds, for Unit-Variance Gaussian Density g.

ε \ c=d	1.0	1.5	2.0	2.5	3.0
0.05	0.0	1.7	2.7	3.5	4.0
0.10	0.0	0.9	2.4	3.0	3.5
0.15	---	0.0	2.0	2.7	3.2
0.20	---	0.0	1.5	2.3	2.8
0.25	---	---	1.2	1.9	2.4
0.30	---	---	0.9	1.6	2.1
0.35	---	---	0.5	1.3	1.8
0.40	---	---	---	0.8	1.5

TABLE II. Upper Bound θ_{max} on θ as Function of c, ε (with c=d) for which Robust LR-Detector is Consistent. (Unit-Variance Gaussian g.)