Spectrum Sensing via Event-triggered Sampling

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***Abstract*—We propose a new framework for cooperative spectrum sensing in cognitive radio networks, that is based on a novel class of non-uniform samplers called the event-triggered samplers, and sequential detection. In the proposed scheme, each secondary user computes its local sensing decision statistic based on its own channel output; and whenever such decision statistic crosses certain predefined threshold values, the secondary user sends one bit (or several bits) information to the fusion center. The fusion center asynchronously receives the bits from different secondary users, based on which it updates the global sensing decision statistic and performs the sequential probability ratio test (SPRT), until reaching a sensing decision. We provide an asymptotic analysis for the above scheme, and under different conditions, we compare it against the cooperative sensing scheme that is based on traditional uniform sampling and sequential detection. Through simulations we show show that the proposed scheme, using even 1 bit, can outperform its uniform sampling counterpart that uses infinite number of bits.**

***Index Terms*—decentralized detection, sequential sensing, asymptotic optimality, event-triggered sampling**

I. Introduction

Spectrum sensing is a fundamental task in cognitive radio systems [1]. In cooperative sensing, there are secondary users (SU) that cooperate to detect the absence/presence of primary users. In this framework, each SU computes a local test statistic using the signal it observes, and sends a summary of it to a fusion center (FC). FC then makes a final sensing decision using the received information from SUs.

The decision mechanism at the FC can be either fixed sample size or sequential. The latter is much more effective in terms of minimizing the decision delay. Sequential Probability Ratio Test (SPRT) is the optimum test for i.i.d. observations among all sequential tests in terms of minimizing the average decision delay [2]. It is the dual of the fixed sample size Neyman-Pearson test.

The majority of the existing works on cooperative sensing assume that SUs sample local test statistics via conventional uniform sampling, and communicate to the FC synchronously. There are a few works in the literature that allow asynchrony among SUs, e.g., [3]–[5]. However, in these papers there is no discussion on the optimality of the proposed methods. In this paper, we develop a new framework for cooperative sensing based on a class of non-uniform samplers called *event-triggered samplers*, in which the sampling times are dynamically determined by the signal to be sampled. In other words, there is no global clock in the proposed scheme as opposed to the conventional uniform sampling case. Thus, asynchronous communication is allowed between SUs and the FC. Moreover, the proposed scheme naturally outputs low-rate information (e.g. 1 bit per sample) without performing any quantization [6]. Both features perfectly match the requirements of cooperative sensing in cognitive radio systems since the control channel for transmitting local statistics has a low bandwith and maintaining synchrony is a demanding task. In addition, in subsequent sections the proposed scheme is theoretically and by simulations shown to outperform the scheme based on conventional uniform sampling.

The remainder of the paper is organized as follows. In section II, the proposed scheme and the competing ones are described. Performances of the schemes are theoretically and numerically analyzed in sections III and IV respectively. Finally, the paper is concluded in section V.

II. Spectrum Sensing via SPRT

A. Centralized sensing

We consider a cognitive radio network where there are K SUs performing spectrum sensing. Let $\{y_t^k\},\ t \in \mathbb{N}$, be the Nyquist-rate sampled discrete-time signal observed by the k-th SU, which transmits a summary of it to the FC. Using the information received at the fusion center from the K SUs, we are interested in deciding between two hypotheses, H_0 and H_1: whether the primary user is present (H_1) or not (H_0). Specifically, every time the fusion center receives new information, it performs a test and either 1) stops accepting more data and decides between the two hypotheses; or 2) postpones its decision until a new data sample arrives from the SUs. When the fusion center stops and selects between the two hypotheses, the whole process is terminated.

Assuming independence across the signals observed by different SUs, we perform the following hypothesis test

$$\begin{array}{ll} \mathsf{H}_0: & \{y_1^k,\dots,y_t^k\} \sim f_0^k,\ k=1,\dots,K \\ \mathsf{H}_1: & \{y_1^k,\dots,y_t^k\} \sim f_1^k,\ k=1,\dots,K \end{array}. \qquad (1)$$

where f_0^k and f_1^k denote the joint probability density function of the received signal by the k-th SU, under H_0 and H_1 respectively.

Each SU computes its own log-likelihood ratio (LLR), L_t^k, and sends it to the FC. Assuming independence across time, we can write L_t^k as follows

$$L_t^k \triangleq \log \frac{f_1^k(y_1^k,\dots,y_t^k)}{f_0^k(y_1^k,\dots,y_t^k)} = \sum_{n=1}^{t} \underbrace{\log \frac{f_1^k(y_n^k)}{f_0^k(y_n^k)}}_{\ell_n^k} = L_{t-1}^k + \ell_t^k \quad (2)$$

FC computes the global LLR, $L_t = \sum_{k=1}^K L_t^k$, and applies SPRT to make a sensing decision.

$$\mathcal{T} = \inf\{t>0 : L_t \notin (-B, A)\}, \quad (3)$$

$$\delta(\mathcal{T}) = \begin{cases} 1, & \text{if } L_{\mathcal{T}} \geq A, \\ 0, & \text{if } L_{\mathcal{T}} \leq -B. \end{cases} \quad (4)$$

Here, $\mathcal{T}$ is the stopping (decision) time and $\delta(\mathcal{T})$ is the decision rule. SPRT minimizes the average decision time, $\mathsf{E}_i[\mathcal{T}],\ i=0,1$ while satisfying the following error probability constraints

$$\mathsf{P}_0(\delta(\mathcal{T})=1) \leq \alpha \qquad \text{and} \qquad \mathsf{P}_1(\delta(\mathcal{T})=0) \leq \beta \quad (5)$$

where $\mathsf{P}_i(\cdot), \mathsf{E}_i[\cdot],\ i=0,1$ denote probability and the corresponding expectation under hypothesis i. Levels $\alpha,\beta \in (0,1)$ are parameters specified by the designer, and the thresholds $A, -B$ are selected so that SPRT satisfies the error probability constraints in (5) with equality.

In other words, at every time instant t, we compare L_t with two thresholds $-B, A$ where $A, B > 0$. As long as L_t stays within the interval $(-B, A)$ we continue taking more data and update L_t. The first time L_t exits $(-B, A)$ we stop (accepting more data) and use the already accumulated information to decide between the two hypotheses $\mathsf{H}_0, \mathsf{H}_1$. At the decision time we decide in favor of H_1 if $L_{\mathcal{T}} \geq A$; and in favor of H_0 if $L_{\mathcal{T}} \leq -B$.

SPRT is optimum in the case of i.i.d. observations, and furthermore for more complex data models it is known to possess strong *asymptotic optimality* properties.

In the centralized scheme given in (3)-(4), the FC has access to all local LLRs. Here, there are two serious practical weaknesses. Firstly, SUs need to send their local LLRs to the FC at the Nyquist-rate of the signal; and secondly, infinite number of bits are required to represent local LLRs which are real numbers. These two problems incur a substantial communication overhead between SUs and the FC. Therefore, in this paper we are interested in *decentralized* schemes in which SUs transmit *low-rate* information to the FC.

B. Decentralized sensing based on uniform sampling

A straightforward way to achieve low-rate transmission is to let each SU uniformly sample its local cumulative LLR with period $T \in \mathbb{N}$; and quantize the sampled values using a finite number of quantization levels. Specifically, during time instants $(m-1)T+1,\dots,mT$, each SU computes its incremental LLR $L_{mT}^k - L_{(m-1)T}^k$ of the observations $y_{(m-1)T+1}^k,\dots,y_{mT}^k$ to obtain

$$\lambda_{mT}^k \triangleq L_{mT}^k - L_{(m-1)T}^k = \sum_{t=(m-1)T+1}^{mT} \ell_t^k \quad (6)$$

where ℓ_t^k is the LLR of the observation y_t^k. It then quantizes λ_{mT}^k into $\tilde{\lambda}_{mT}^k$ using a finite number $\tilde{r}$ of quantization levels. Assuming $\max_{k,t}|\ell_t^k| \leq \phi < \infty$, i.e. $|\lambda_{mT}^k| \leq T\phi$, and using a conventional mid-riser quantizer we write $\tilde{\lambda}_{mT}^k$ as follows

$$\tilde{\lambda}_{mT}^k = -T\phi + \frac{T\phi}{\tilde{r}} + \left\lfloor \frac{\tilde{r}(\lambda_{mT}^k + T\phi)}{2T\phi} \right\rfloor \frac{2T\phi}{\tilde{r}}. \quad (7)$$

The index of the quantization level is then transmitted to the FC using $s = \log_2 \tilde{r}$ bits.

The FC receives the quantized information from all SUs *synchronously*, and updates the approximation of the global running LLR based on the information received,i.e.,

$$\tilde{L}_{mT} = \tilde{L}_{(m-1)T} + \sum_{k=1}^K \tilde{\lambda}_{mT}^k. \quad (8)$$

Then the FC, using $\tilde{L}_t, \tilde{A}, -\tilde{B}$, employs the scheme in (3)-(4) to obtain $\tilde{\mathcal{T}}$ and $\tilde{\delta}(\tilde{\mathcal{T}})$. Again, the two thresholds $\tilde{A}, -\tilde{B}$ are selected to satisfy the error probability constraints in (5) with equality. We call this scheme the *Quantized SPRT* and denote it as Q-SPRT.

C. Decentralized sensing based on event-triggered sampling

In this paper, we achieve low-rate transmission via *level-triggered* sampling which is a simple form of event-triggered sampling. A local LLR process is sampled whenever it crosses a predetermined level, so sampling times are determined by the LLR process to be sampled and thus random. This constitutes the fundamental difference between level-triggered sampling and uniform sampling. In level-triggered sampling, sampling times (or inter-sampling intervals) carry most of the information regarding the sampled value. However, in uniform sampling all information is encoded into the sampled values; and sampling times bear no information.

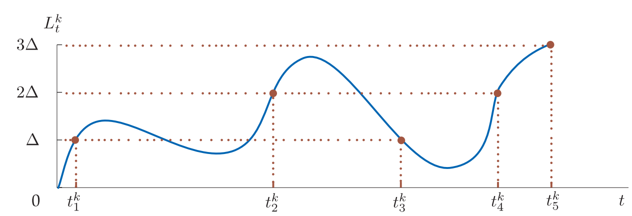

Fig. 1. A realization of a local LLR process and its sampling times when level-triggered sampling with sampling level Δ is applied.

Using level-triggered sampling with uniform sampling levels Δ, each SU, as shown in Fig. 1 samples its local cumulative LLR process $\{L_t^k\}$ at a sequence of random times $\{t_n^k\}$, which is particular to each SU. In other words we do not assume any type of synchronization in sampling and therefore communication. The corresponding sequence of samples is $\{L_{t_n^k}^k\}$, and the sequence of sampling times is recursively defined as follows

$$t_n^k \triangleq \inf\left\{t > t_{n-1}^k : L_t^k - L_{t_{n-1}^k}^k \notin (-\Delta, \Delta)\right\} \quad (9)$$

where Δ is a constant, and $t_0^k = 0,\ L_0^k = 0$. Then, each SU simply sends the information of the level that is crossed (upper or lower) using 1 bit. If $\lambda_n^k \triangleq L^k_{t^k_n} - L^k_{t^k_{n-1}} \geq \Delta$, then $b_n^k = 1$ is sent to the FC. Similarly, for $\lambda_n^k \leq -\Delta$, the information bit $b_n^k = -1$ is sent to the FC. In other words, we have $b_n^k = \text{sign}(\lambda_n^k)$. Using the information bits $\{b_j^k\}_{j=1}^n$, at time t_n^k, L_t^k is approximately written as follows

$$\hat{L}^k_{t^k_n} = \sum_{j=1}^{n} b_j^k \Delta = \hat{L}^k_{t^k_{n-1}} + b_n^k \Delta. \tag{10}$$

Note that we will have $\hat{L}^k_{t^k_n} = L^k_{t^k_n}$ if λ_n^k perfectly hits the thresholds $\Delta, \ -\Delta$. This is the case when SUs observe continuous-time and continuous-path signals. Note also that each SU, in fact, performs a local SPRT with thresholds Δ and $-\Delta$.

The FC asynchronously receives information bits $\{b_n^k\}_k$ from different SUs, and computes the approximate global running LLR as follows

$$\hat{L}_t = \sum_{k-1}^{K} \hat{L}_t^k = \Delta \sum_{k=1}^{K} \sum_{n: t_n^k \leq t} b_n^k. \tag{11}$$

Specifically, the FC adds Δ to the approximate global LLR whenever it receives a 1; and subtracts Δ whenever it receives a -1. Finally, the FC, using $\hat{L}_t$, applies SPRT given in (3)-(4) with thresholds $\hat{A}$ and $-\hat{B}$ to make a final sensing decision $\hat{\delta}(\hat{\mathcal{T}})$. $\hat{A}, -\hat{B}$ are selected to satisfy the error probability constraints in (5) with equality. We call this scheme the *Level-triggered SPRT* and denote it as LT-SPRT.

D. Randomized Overshoot Quantization for LT-SPRT

With continuous-time observations at the SUs, LT-SPRT using only 1 bit exhibits a strong (order-2) asymptotic optimality property (see Section III) due to having exact recovery, i.e. $\hat{L}^k_{t^k_n} = L^k_{t^k_n}$, at any sampling time t_n^k. Therefore, in this case there is no need to use additional bits to improve the performance of LT-SPRT.

On the other hand, with discrete-time observations at the SUs λ_n^k can overshoot Δ or undershoot $-\Delta$. Consequently, the difference $|L^k_{t^k_n} - \hat{L}^k_{t^k_n}|$ is an important source of performance degradation. In order to compensate for this we use additional bits to quantize the over(under)shoot $q_n^k \triangleq |\lambda_n^k| - \Delta$.

The overshoot q_n^k is smaller than the LLR of the last observation $\ell^k_{t^k_n}$ which is assumed to be bounded by ϕ, i.e., $q_n^k \leq \ell^k_{t^k_n} \leq \phi$. We uniformly divide the interval $[0, \phi]$ into $\hat{r}$ subintervals, and send either lower or upper end of the subinterval that q_n^k falls into according to a specific randomization rule. Specifically, we, using $s = \log_2(\hat{r}+1)$ bits, send the index of either lower end with probability p or upper end with probability $1-p$, i.e.,

$$\hat{q}_n^k = \begin{cases} \lfloor \frac{q_n^k \hat{r}}{\phi} \rfloor \frac{\phi}{\hat{r}}, & \text{prob. } p = \frac{\exp(-q_n^k + (\lfloor \frac{q_n^k \hat{r}}{\phi} \rfloor + 1)\frac{\phi}{\hat{r}}) - 1}{\exp(\frac{\phi}{\hat{r}}) - 1} \\ (\lfloor \frac{q_n^k \hat{r}}{\phi} \rfloor + 1)\frac{\phi}{\hat{r}}, & \text{prob. } 1-p = \frac{1 - \exp(-q_n^k + \lfloor \frac{q_n^k \hat{r}}{\phi} \rfloor \frac{\phi}{\hat{r}})}{1 - \exp(-\frac{\phi}{\hat{r}})}. \end{cases} \tag{12}$$

The randomization probability p is selected to simplify the asymptotic optimality analysis. More details on the selection of p can ben found in [7].

The FC now receives the information $(b_n, \hat{q}_n)$ at time t_n from some SU, and updates its approximate global LLR as follows

$$\hat{L}_{t_n} = \hat{L}_{t_{n-1}} + b_n(\Delta + \hat{q}_n). \tag{13}$$

This scheme will be called *Randomized Level-triggered SPRT*, and denoted as RLT-SPRT.

III. Performance Analysis

In this section, we provide some results for the asymptotic optimality analysis on the stopping (decision) time of Q-SPRT and RLT-SPRT. For our comparisons we will use the notion of asymptotic optimality. A decentralized scheme with stopping time $\mathcal{S}$ is said to be order-1 asymptotically optimal if the ratio $\frac{\mathsf{E}_i[\mathcal{S}]}{\mathsf{E}_i[\mathcal{T}]}$ tends to 1, i.e.,

$$\frac{\mathsf{E}_i[\mathcal{S}]}{\mathsf{E}_i[\mathcal{T}]} = 1 + o_{\alpha,\beta}(1)^1 \tag{14}$$

as the target error probabilities tend to zero, i.e., $\alpha, \beta \to 0$. Here, $\mathcal{T}$ is the stopping time of the optimum centralized scheme. Similarly, a decentralized scheme with stopping time $\mathcal{S}$ is said to be order-2 asymptotically optimal if the difference $\mathsf{E}_i[\mathcal{S}] - \mathsf{E}_i[\mathcal{T}]$ remains bounded, i.e.,

$$\mathsf{E}_i[\mathcal{S}] - \mathsf{E}_i[\mathcal{T}] = O(1) \tag{15}$$

as $\alpha, \beta \to 0$. Note that order-2 asymptotic optimality implies order-1 asymptotic optimality, but not vice versa. If SUs observe continuous-time, continuous-path signals, it is known that RLT-SPRT enjoys *order-2* optimality by using only 1 bit [6], whereas Q-SPRT with a fixed number of bits cannot achieve any type of asymptotic optimality [8].

If SUs observe discrete-time observations, then the analyses become more involved due to the overshoot effect. In this section, we will show that RLT-SPRT can attain order-2 asymptotic optimality in the discrete-time case.

We start by presenting a lemma that quantifies the performance of the optimum centralized scheme.

Lemma 1. *Assuming that the two error probabilities $\alpha, \beta \to 0$ at the same rate, the centralized SPRT, satisfies*

$$\begin{aligned} \mathsf{E}_0[\mathcal{T}] \geq & \frac{1}{K \mathsf{I}_0} \mathcal{H}(\alpha, \beta) = \frac{|\log \beta|}{K \mathsf{I}_0} + o_\beta(1) \\ \mathsf{E}_1[\mathcal{T}] \geq & \frac{1}{K \mathsf{I}_1} \mathcal{H}(\beta, \alpha) = \frac{|\log \alpha|}{K \mathsf{I}_1} + o_\alpha(1) \end{aligned} \tag{16}$$

where $\mathcal{H}(x, y) = x \log \frac{x}{1-y} + (1-x) \log \frac{1-x}{y}$; and $\mathsf{I}_i = \frac{1}{K}|\mathsf{E}_i[L_1]|$, $i = 0, 1$ are the average Kullback-Leibler information numbers of the process $\{L_t\}$ under the two hypotheses.

Proof: It should be noted that these inequalities become equalities in the continuous-time continuous-path case. The proof can be found in [9, Page 21]. ■

[1] $o_x(1)$ denotes a quantity that becomes negligible with x, and $o_{x,y}(1)$ denotes a quantity that becomes negligible with x or with y or with both.

The following two theorems state the asymptotic analysis results on the stopping times of Q-SPRT and RLT-SPRT respectively.

Theorem 1. *Assuming that the two error probabilities $\alpha,\beta \to 0$ at the same rate, and that the number $\tilde{r}$ of quantization levels increases with α,β, then the performance of Q-SPRT, $\tilde{\mathcal{T}}$, as compared to the optimum centralized SPRT, $\mathcal{T}$, satisfies*

$$\begin{aligned}\mathsf{E}_0[\tilde{\mathcal{T}}]-\mathsf{E}_0[\mathcal{T}] \leq & \frac{\phi}{K\mathsf{I}_0^2}\frac{|\log\beta|}{2^{s-1}}\{1+o_s(1)\}+\\ & T\frac{\phi}{\mathsf{I}_0}\{1+o_s(1)\}+o_\beta(1)\\ \mathsf{E}_1[\tilde{\mathcal{T}}]-\mathsf{E}_1[\mathcal{T}] \leq & \frac{\phi}{K\mathsf{I}_1^2}\frac{|\log\alpha|}{2^{s-1}}\{1+o_s(1)\}+\\ & T\frac{\phi}{\mathsf{I}_1}\{1+o_s(1)\}+o_\alpha(1).\end{aligned} \tag{17}$$

Proof: The proof can be found in [7, Appendix A]. ∎

Theorem 2. *Assuming that the two error probabilities $\alpha,\beta \to 0$ at the same rate, and that the number $\hat{r}$ of quantization levels increases with α,β, then the performance of RLT-SPRT, $\hat{\mathcal{T}}$, as compared to the optimum centralized SPRT, $\mathcal{T}$, satisfies*

$$\begin{aligned}\mathsf{E}_0[\hat{\mathcal{T}}]-\mathsf{E}_0[\mathcal{T}] \leq & \frac{1}{T}\frac{\phi}{K\mathsf{I}_0^2}\frac{|\log\beta|}{\max\{2^{s-1}-1,1\}}\{1+o_{T,s}(1)\}+\\ & T+\frac{\phi}{\mathsf{I}_0}+o_{T,s}(1)+o_\beta(1),\\ \mathsf{E}_1[\hat{\mathcal{T}}]-\mathsf{E}_1[\mathcal{T}] \leq & \frac{1}{T}\frac{\phi}{K\mathsf{I}_1^2}\frac{|\log\alpha|}{\max\{2^{s-1}-1,1\}}\{1+o_{T,s}(1)\}+\\ & T+\frac{\phi}{\mathsf{I}_1}+o_{T,s}(1)+o_\alpha(1).\end{aligned} \tag{18}$$

Proof: The proof is presented in [7, Appendix B]. ∎

In order to write (17) and (18), we ensure that the average rate of received messages by the FC is the same for Q-SPRT and RLT-SPRT. In Q-SPRT, the FC every T units of time synchronously receives K messages from all SUs, thus the average message rate is $\frac{K}{T}$. In RLT-SPRT, we consider the time interval $[0,t]$ and denote with $\mathcal{N}_t$ the total number of messages received by the FC up to time t. If we also denote with $\mathcal{N}_t^k$ the number of messages sent by the k-th SU until t, then we have $\mathcal{N}_t=\sum_{k=1}^K \mathcal{N}_t^k$. For the average message rate, we need to compute the following limit

$$\begin{aligned}\lim_{t\to\infty}\frac{\mathcal{N}_t}{t} &= \lim_{t\to\infty}\sum_{k=1}^K \mathcal{N}_t^k \\ &= \sum_{k=1}^K \lim_{t\to\infty}\frac{1}{\frac{1}{\mathcal{N}_t^k}(\sum_{n=1}^{\mathcal{N}_t^k} t_n^k - t_{n-1}^k)+\frac{1}{\mathcal{N}_t^k}(t-t_{\mathcal{N}_t^k}^k)} \\ &= \sum_{k=1}^K \frac{1}{\mathsf{E}_i[t_1^k]},\end{aligned} \tag{19}$$

where we recall that $\{t_n^k\}$ is the sequence of sampling times at the k-th SU and for the last equality we used the Law of Large Numbers since when $t\to\infty$ we also have $\mathcal{N}_t^k\to\infty$. Consequently we need to select Δ so that $\sum_{k=1}^K\frac{1}{\mathsf{E}_i[t_1^k]}=\frac{K}{T}$. In [7], it is shown that if we set $\Delta\tanh(\frac{\Delta}{2})=T\mathsf{I}_i$, then $\sum_{k=1}^K\frac{1}{\mathsf{E}_i[t_1^k]}\simeq\frac{K}{T}$.

Although there is a definite resemblance between (17) and (18), the factor $\frac{1}{T}$ in the first term of the right side in (18) produces significant performance gains. Since T is the communication period, and we are in discrete time, we have $T\geq 1$. Actually for the practical problem of interest $T\gg 1$, which suggests that the first term in RLT-SPRT is smaller by a factor T, which can be large.

It is easy to see that *order-2* asymptotic optimality is achievable by keeping T constant and setting the number of bits $s=1+\log_2|\log\alpha|$ in (17), and $s=1+\log_2|\log\alpha|-\log_2 T$ in (18). Even though the two expressions for the number of bits are of the same magnitude, RLT-SPRT needs significantly less bits than Q-SPRT to assure order-2 asymptotic optimality for all practical purposes.

On the other hand, for fixed s RLT-SPRT attains *order-1* asymptotic optimality by letting $T\to\infty$ with a slower rate than the rate $|\log\alpha|\to\infty$, i.e., $\frac{T}{|\log\alpha|}\to 0$. However, Q-SPRT cannot enjoy any type of asymptotic optimality by just controlling T due to the missing factor $\frac{1}{T}$ in (17).

IV. Simulation Results

In this section, we provide simulation results on the asymptotic optimality properties of the decentralized schemes to verify the theoretical findings in Section III. We consider a cognitive radio system with two SUs, i.e., $K=2$. In Q-SPRT, the sampling period is set to $T=4$. In RLT-SPRT, we adjust the local threshold Δ so that the average rate of received messages by the FC matches that of Q-SPRT. Energy detection is used as the spectrum sensing method at the SUs (Gaussian detection and correlation detection yield similar results). For the energy detector, we set the receiver SNR for each SU to 5 dB and vary the error probabilities α and β together between 10^{-1} and 10^{-10}. All results are obtained by averaging 10^4 trials and using importance sampling to compute probabilities of rare events. In the subsequent figures, average sensing delay performances under H_1 are plotted against error probabilities $\alpha=\beta$.

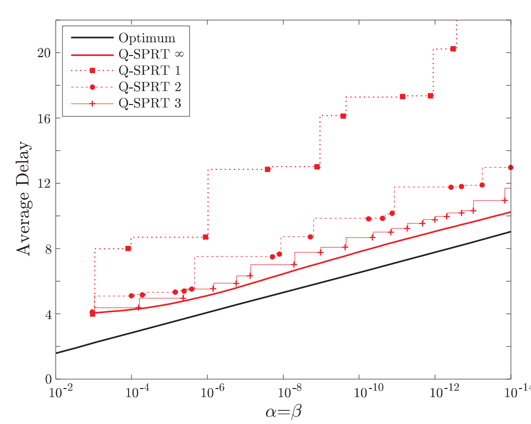

Fig. 2. Average detection delay vs error probabilities (α,β) for optimum centralized and Q-SPRT with 1,2,3,∞ number of bits.

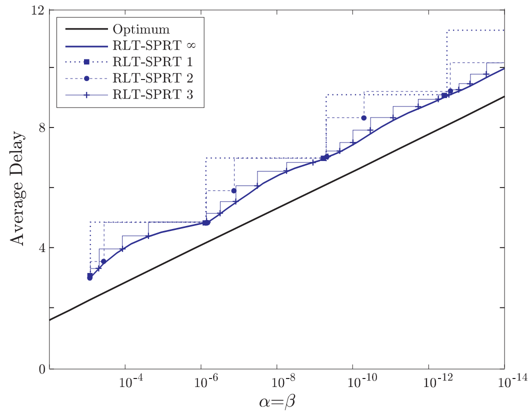

Fig. 3. Average detection delay vs error probabilities (α, β) for optimum centralized and RLT-SPRT with 1,2,3,∞ number of bits.

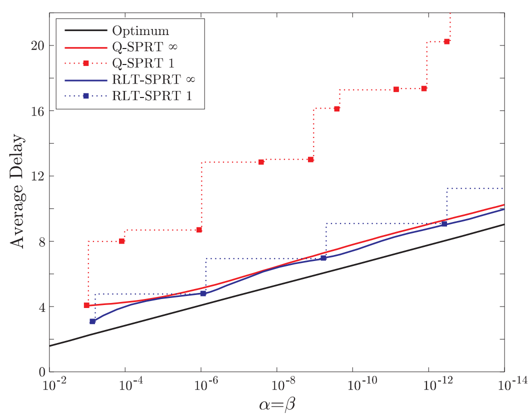

Fig. 4. Average detection delay vs error probabilities (α, β) for optimum centralized, Q-SPRT and RLT-SPRT with 1,∞ number of bits.

Fig. 2 and Fig. 3 illustrate the asymptotic performances of Q-SPRT and RLT-SPRT with $1, 2, 3$ and ∞ number of bits; and Fig. 4 compares the 1-bit and the ∞-bit cases of both schemes with the optimum centralized scheme. Our first interesting result is the fact that by using a finite number of bits we can only achieve a discrete number of error rates. Specifically, if a finite number of bits is used to represent local incremental LLR packages, then there is a finite number of possible values to update the global running LLR (e.g. for one bit we have $\pm\Delta$). This suggests that any threshold between two consecutive LLR values will produce the same error probability. Consequently, only a discrete set of error probabilities (α, β) are achievable. The case of infinite number of bits corresponds to the best achievable performance for Q-SPRT and RLT-SPRT. Having their performance curves parallel to that of the optimum centralized scheme, the ∞-bit case for both Q-SPRT and RLT-SPRT exhibits order-2 asymptotic optimality. Recall that both schemes can enjoy order-2 optimality if the number of bits tends to infinity with a rate of $\log|\log\alpha|$.

It is notable that the performance of RLT-SPRT with a small number of bits is very close to that of ∞-bit RLT-SPRT at achievable error rates. For instance, the performance of 1-bit case coincides with that of ∞-bit case, but only at a discrete set of points as can be seen in Fig. 3. However, we do not observe this feature for Q-SPRT. Q-SPRT with a small number of bits (especially one bit) performs significantly worse than ∞-bit case Q-SPRT as well as its RLT-SPRT counterpart. Finally, it is a striking result that 1-bit RLT-SPRT is superior to ∞-bit Q-SPRT at its achievable error rates, which can be seen in Fig. 4.

V. Conclusion

We have proposed and analyzed a new spectrum sensing scheme for cognitive radio networks. The proposed scheme is based on *level-triggered sampling* which is a non-uniform sampling technique that naturally outputs 1 bit information without performing any quantization, and allows SUs to communicate to the FC asynchronously. Therefore, it is truly decentralized, and ideally suits the cooperative spectrum sensing in cognitive radio networks.

The proposed scheme (RLT-SPRT) achieves order-2 asymptotic optimality in both continuous-time and discrete-time cases. With discrete-time observations at SUs, it needs significantly less number of bits than its uniform sampling counterpart (Q-SPRT) to attain order-2 asymptotic optimality. With a fixed number of bits, unlike Q-SPRT, our scheme can also attain order-1 asymptotic optimality when the average communication period tends to infinity with a rate slower than that of $|\log\alpha|$. Simulation results demonstrated the fact that RLT-SPRT, using 1 bit, performs significantly better than 1-bit Q-SPRT, and even better than ∞-bit Q-SPRT at its achievable error rates.

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