THE LOCAL METHOD APPLIED TO THE ROBUST DETECTION OF CHANGES IN THE POLES OF A POLE-ZERO SYSTEM

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In this chapter, we investigate a non classical change detection problem, for which there is a strong coupling effect between nuisance parameters and parameters to be monitored. As maximum likelihood methods cannot be used in this case, we derive a so-called instrumental statistics which, together with a local testing approach, gives a new test of χ^2 type. The extension of this test for the problem of diagnosis is also described.

Only the scalar case is investigated here. The extension of the proposed tests for vector signals is currently under study and may be used, for example, as a solution to the problem of vibration monitoring for offshore platforms.

I - INTRODUCTION

The purpose of this chapter is the presentation of a possible solution to a change detection problem such that :

- the value, after change, of the parameters of interest (or monitored parameters) is unknown ;

- the change is possibly small ;

- there is a strong coupling effect between the nuisance parameters and the monitored parameters ;

- in addition to the *global* testing problem (i.e. decide whether the parameters, taken in their entire set, have changed), the *diagnosis* of which special features have changed is also of interest.

The solutions we propose is based upon an "instrumental" statistics, which is not extracted from the likelihood of the observations ; for deriving the decision function, we use thelocal approach for testing statistical hypotheses which has been presented and investigated in chapters 4 [5] and 7 [12].

Before presenting the solution, let us describe further the problem at hand. In some practical situations, such as vibration monitoring, it is of interest to detect changes in the characteristics of a system without knowing the characteristics of its excitation, which can be nonstationary; for example, in vibration monitoring, one wishes to detect changes in vibrating characteristics of systems subject to nonstationary unknown excitation (swell, wind, earthquakes...). In such cases, the change detection problem can be formulated as follows ; using an ARMA model with (highly) nonstationary unknown MA coefficients to model the excitation [13], detect a change in the AR part (assumed stationary) and, if possible, determine which AR coefficients or which poles and modes have changed (i.e. diagnosis problem).

Because of the time scales of our application on offshore platforms, namely because the sampling rate is very much higher than the change rate, we describe here an off-line change detection procedure, which is in fact a statistical hypotheses test procedure : we decide whether a new ($\underset{-}{\sim}$ half an hour) record of measurement signals (accelerometers or strain gauges), containing typically 40000 sample points, behaves in conformity with a reference model of the structure identified on a previous (several months before) record. But, it should be clear (see, for example, the discussion in chapter 4 [5]) that this decision procedure we propose may be implemented in a competely on-line framework : for example, using the GLR method [5] or the so-called two models approach of chapter 6 [1].

As discussed in chapter 7 [12], because the model after change is unknown and because simple procedures are of interest, the use of the maximum likelihood (of the observations) test, or first version of CSA, is not possible. Moreover, as small changes are to be detected, the use of the so-called local approach [8] is more convenient ([5], [12]). Furthermore, as far as nuisance parameters are concerned, namely the MA coefficients in our ARMA model, it has been seen in chapter 7 [12] that the local approach applied on the likelihood of the observations is not convenient because of the strong coupling effect between the AR and the MA parameters (the Fisher information matrix is not block diagonal). On the other hand, as the unknown MA parameters are not only nonstationary but may also be subject to jumps, standard elimination methods for nuisance parameters [3], such as for example maximization of the likelihood with respect to the unknown parameters, seem to be of no help for the present test problem.

For all these reasons, the test statistics we propose is derived using the local approach not applied to the likelihood of the observations, but rather to an instrumental statistics U. This statistics comes from the so-called instrumental variable identification method [14] which has been proved [4] to provide with consistent estimates of the AR part without knowing (or using estimates of) the varying MA part. The central limit theorem is shown to hold [10] for this statistics U, under both hypotheses, null H_0 (i.e. no change) and local alternative H_1 (i.e. small change). This gives a χ^2-type global test, without diagnosis. Using the effect of specific parameters changes (poles or vibrating modes for example), on the mean of U under H_1, specific tests for monitoring vibrating modes separately may be designed.

In this paper, we investigate only the scalar case. The extension to the vector case will be reported later.

II - DETECTION OF CHANGES IN THE AR PART OF AN ARMA MODEL WITH NONSTATIONARY UNKNOWN MA COEFFICIENTS

Let us consider the following model :

$$y_t = \sum_{i=1}^{p} a_i \, y_{t-i} + \sum_{j=0}^{q} b_j(t) \; e_{t-j} \qquad (1)$$

where $(e_t)_t$ is a gaussian white noise with constant variance σ_e^2 , and where the unknown MA coefficients $b_j(t)$ are time varying and possibly subject to jumps. The problem to be solved is the (off-line) detection of changes in the AR parameters (a_i). Let us first recall the main result concerning the identification problem, because as stated in the INTRODUCTION, it is the starting point of our detection procedure.

1. - Identification of the AR coefficients without knowing the non-stationary MA coefficients

Assume that a single record $(y_1,\ldots,y_s)$ of the process (y_t) is available. The so-called instrumental variable method for identification has been recently proved [4] to provide with consistent estimates of the autoregressive parameters in the present framework. More precisely, let :

$$\mathcal{H}_{p,N-1}(s) = \begin{bmatrix} R_{q-p+1}(s) & R_{q-p+2}(s) & \cdots & R_{q+1}(s) & R_{q+2}(s) & \cdots & R_{N+q-p}(s) \\ R_{q-p+2}(s) & & & & & & \vdots \\ R_{q-p+3}(s) & & & & & & \vdots \\ \vdots & & & & & & \vdots \\ R_{q+1}(s) & R_{q+2}(s) & & & & \cdots & R_{N+q}(s) \end{bmatrix} \qquad (2)$$

the $(p+1) \times N$ empirical Hankel matrix of the process $(y_t)_{0 \leqslant t \leqslant s}$ where

$N \geqslant p$ is the number of instruments and

$$R_k(s) = \sum_{t=0}^{s-k} y_{t+k} \; y_t \quad (k \geqslant 0) \quad .$$

Then the least squares solution $(\hat{a}_p(s), \hat{a}_{p-1}(s), \ldots, \hat{a}_1(s))$ of the equation :

$$(-a_p \; -a_{p-1} \; \ldots \; -a_1 \; 1) \; \mathcal{H}_{p,N-1}(s) = 0$$

is a consistent estimate of true vector parameter :

$$\theta = (a_p \; a_{p-1} \ldots a_1)' \qquad (3)$$

of model (1). See [4] for a complete proof and precise statement of the consistency result. This result does not require any stationarity assumption about the moving average parameters $b_j(t)$. In this sense, this identification method of the AR part may be thought as being robust with respect to the unknown MA part.

2. - The change detection problem

The use of standard observation-based likelihood ratio techniques for solving this problem would require either an identification of the MA coefficients $b_j(t)$ using for example a forgetting factor, or maximization or integration of the likelihood with respect to a prior distribution of these unknown parameters [3]. Because of their highly varying features (related for example to the shock or turbulence effects of the sea on an offshore platform), these approaches do not seem to be appropriate. (Remember also that in [6] Bohlin assumed that convenient values of the MA coefficients were available).

Moreover, the Fisher information matrix of an ARMA model is not block diagonal : there is an interaction between the AR and the MA coefficients. In other words, there is a coupling effect between the detection of changes in poles or zeroes, and therefore it is not convenient to use (local) likelihood methods ([8], [9], [5], [12]) for detecting changes on poles when the zeroes have to be viewed as nuisance parameters.

Keeping in mind the "robustness" properties of the identification procedure with respect to the nuisance parameters, we propose the following off-line change detection procedure. Let us now assume that a "reference" model parameter $\theta_0 = (a_p^0 \dots a_1^0)'$ has been estimated on a record of signals y, and let us consider the following problem : given a new record of signals y, decide whether they follow the same model or not. The solution we propose is the following : compute again the empirical Hankel matrix $\mathcal{H}_{p,N}$ corresponding to this new record, and look at the "size" of the vector U_N

defined by : $U'_N(s) = (-a_p^0 \ldots -a_1^0 \;\; 1) \; \mathcal{H}_{p,N-1}(s)$ (4)

If there has been no change in the AR part, this U vector should be "close" to zero ; in case though of a change in the AR parameters, this U vector should be "significantly" different from zero.

Let us rewrite $U_N(s)$, in a numerically more efficient way, as :

$$U_N(s) = \sum_{t=q+N}^{s} w_t \, Z_t \tag{5}$$

where : $w_t = y_t - a_1 \, y_{t-1} - \ldots - a_p \, y_{t-p}$ (6)

is the "moving average part", and :

$$Z_t = (y_{t-q-1} \; y_{t-q-2} \ldots y_{t-q-N})' \tag{7}$$

Under the hypothesis of no change (i.e. θ_0 still represents the AR part of the actual process), Z_t is *orthogonal* to w_t and the covariance matrix of U is :

$$\Sigma_N(s) = \sum_{t=q+N}^{s-q} \; \sum_{i=-q}^{q} \; \mathbb{E}_{\theta_0} \, (w_t \, w_{t-i} \, Z_t \, Z'_{t-i}) \tag{8}$$

because, for $|t-r| \geqslant q+1$: $\mathbb{E}_{\theta_0} \; (w_t \, w_r \, Z_t \, Z'_r) = 0$.

Finally, let $\hat{\Sigma}_N(s)$ be the following matrix :

$$\hat{\Sigma}_N(s) = \sum_{t=q+N}^{s-q} \; \sum_{i=-q}^{q} \; w_t \, w_{t-i} \, Z_t \, Z'_{t-i} \tag{9}$$

Despite the fact that the process y_t, and thus Z_t, is *nonstationary*, it turns out that the two following theorems hold [10] :

i) Nonstationary law of large numbers

$\hat{\Sigma}_N$ is a consistent estimate of Σ_N, namely :

$$\Sigma_N^{-1}(s) \quad \hat{\Sigma}_N(s) \quad \xrightarrow[s\to\infty]{} \quad I_N \tag{10}$$

under both null hypothesis, i.e. the set of AR parameters is θ_0, and local alternative hypothesis, i.e. the set of AR parameters is $\theta_0 + \frac{\delta\theta}{\sqrt{s}}$, where $\delta\theta$ is fixed.

ii) Central limit theorem

Under the probability law $\mathbb{P}_{\theta_0}$, we have :

$$\hat{\Sigma}_N(s)^{-\frac{1}{2}} \quad . \quad U_N(s) \quad \xrightarrow[s\to\infty]{} \quad \mathcal{N}(0,I_N) \tag{11}$$

and under the "small" change hypothesis $\mathbb{P}_{\theta_0+\frac{\delta\theta}{\sqrt{s}}}$ we have :

$$\hat{\Sigma}_N(s)^{-\frac{1}{2}} \quad . \quad (U_N(s) - \mathcal{H}_{N-1,p-1} \quad \frac{\delta\theta}{\sqrt{s}}) \quad \xrightarrow[s\to\infty]{} \quad \mathcal{N}(0,I_N) \tag{12}$$

Basically, the proofs are based upon extensive uses of various limit theorems for martingales.

In other words, the use of the local approach for detecting changes ([5], [8], [12]) reduces the original problem to the problem of detecting a change in the mean value of a gaussian process.

Let us consider the generalized likelihood ratio ([8], [16]) as the decision rule for this new problem. Maximization with respect to all possible "magnitudes" of changes $\delta\theta$ is straightforward, and lead to the following χ^2 test :

$$t_0 = U_N' \quad \hat{\Sigma}_N^{-1} \mathcal{H}'_{p-1,N-1} \left(\mathcal{H}_{p-1,N-1} \quad \hat{\Sigma}_N^{-1} \mathcal{H}'_{p-1,N-1} \right)^{-1} \mathcal{H}_{p-1,N-1} \quad \hat{\Sigma}_N^{-1} \quad U_N \tag{13}$$

In [2], special cases are investigated, namely the AR case (q=0) for which (13) is nothing but the third version of the CSA of Nikiforov [12], and the ARMA (p,p-1) case which naturally arises from state space models without observation noise (used for offshore platforms).

Finally, let us emphasize that this test may be extended to the vector case. The optimization of the power of this test may be used as a criterion for optimum sensor location design. These are currently under study.

3 - Detection with diagnosis

We now consider the problem of detecting changes in the AR part, with diagnosis upon which AR coefficients or which poles have changed, and still without knowing the nonstationary zeroes. Even in the stationary AR case, this diagnosis problem does not seem to be standard ; as far as we know, the only approach which has been investigated is the so-called multiple model approach described in chapter 2 [16] and [15]. We describe here a sensitivity method which looks for changes constrained into a sub-space ; another approach (decoupling method) is investigated in [2].

We have seen in the previous paragraph that a possible solution to the problem of detecting changes in the AR parameters without knowing the MA ones, is to solve the equivalent gaussian testing problem for the "instrumental" statistics U . Remember (11) and (12) which summarize the nonstationary central limit theorem.

The basic idea underlying the sensitivity method is to take into account the effect $\delta\theta$ of changes of interest (for example on separate poles) on the θ parameter (3) *and* to use the same likelihood ratio approach based upon the U vector.

We will make an extensive use of the following general result. Assume U is (asymptotically) distributed as $\mathcal{N}(0,\Sigma)$ under H_0 and as $\mathcal{N}(\mu,\Sigma)$ under H_1. For testing $\mu = 0$ against $\mu \neq 0$, one has to compute :

$$U' \Sigma^{-1} U \tag{14}$$

For testing $\mu = 0$ against $\mu \in$ Range (A), where A is a given low rank matrix, compute :

$$U' \Sigma^{-1} A (A' \Sigma^{-1} A)^{-1} A' \Sigma^{-1} U \tag{15}$$

which is nothing but the maximum value, with respect to ν, of the log likelihood ratio between H_0 and H_1 with $\mu = A\nu$.

Describing more precisely the diagnosis problem, let ψ be the m-dimensional set of the "free" parameters to be monitored, and ψ_0 the set of their nominal values. Then, changes $\delta\psi$ in these free parameters induce changes in the AR parameters $\delta\theta$ given by :

$$\delta\theta = f(\delta\psi)$$

where f is a non linear differentiable function.

Let $J = f'(\psi_0)$ be the pxm Jacobian matrix :

$$J = \left(\frac{\partial a_i}{\partial \psi_j}\right)_{\{\psi_j\} = \psi_0} \tag{16}$$

A first order approximation leads to :

$$\delta\theta = J\ \delta\psi \ ;$$

in other words the changes on the AR parameters are constrained to the subspace Range (J). The corresponding diagnosis test is nothing but (15) with

$$A = \mathcal{H}'_{p-1,N-1} \quad J_{p,m} \quad .$$

For example, if the diagnosis problem of interest is to monitor eigenfrequencies ω_j, the corresponding tests are :

$$G_j' \quad W_j^{-1} \quad G_j \gtrless \lambda \tag{17}$$

where

$$\begin{cases} G_j = \dfrac{\partial \theta'}{\partial \omega_j} \quad \mathcal{H}' \Sigma^{-1} \; U \\ W_j = \dfrac{\partial \theta'}{\partial \omega_j} \quad \mathcal{H}' \; \Sigma^{-1} \quad \mathcal{H} \dfrac{\partial \theta}{\partial \omega_j} \end{cases} \tag{18}$$

and where $\frac{\partial \theta}{\partial \omega_j}$ may be computed off-line (numerical values of the derivatives are computed at the nominal poles). The computation of the Jacobian matrix J in this special case may be found in [2] and [7].

The advantage of this approach is that it allows the separate monitoring of as many poles or subsets of poles as desired, without a priori knowing which poles will actually change. The main drawback is that there may exist a coupling effect between the poles to be monitored ; namely all the separate tests can be non zero even if only one pole has actually moved. But it will be shown in section III that the diagnosis decision is nevertheless correct, in most of the cases.

Again, these tests may be extended to the vector case.

III - SOME NUMERICAL RESULTS

The experiments which have been done are highly motivated by the fact that, in view of the application to vibration monitoring, we are interested in detecting *small* changes in eigenfrequencies. "Small" here means one per cent ; in other words, according to the location of the corresponding poles, the "observable" change, namely the change in the AR coefficients, may be less than four per thousand.

We have chosen models ofeven order, with pairs of complex conjugate poles, of the form :

$$(\rho_j e^{i\omega_j} , \rho_j e^{-i\omega_j})$$

and studied changes in one or more ω_j. In most cases, the ρ_j are equal, but the influence of these parameters has also been studied. We will show that a fixed pole close to the unit circle can prevent the diagnosis, and even the "global" detection of a change in a second pole far from the unit circle. The models which have been used are shown in table 1. For each experiment, the numerical values of the test statistics are computed under both H_0 (i.e. the actual model is the reference model) and H_1 (i.e. the actual model is the changed model).

Table 2 gives the values of the global test and the sensitivity test for diagnosis in the special case of *stationary AR signals*. The reason for considering this case is the analysis of the coupling effect during diagnosis mentioned in section II.3).

Table 3 gives the corresponding results for the ARMA (p,p-1) case, where the moving average coefficients are piecewise constant (lengths of intervals randomly chosen). Other experiments can be found in [2]

First results on real (offshore platforms) scalar signals are also encouraging.

IV - CONCLUSION

The problem of detection and diagnosis of changes in the poles of a pole-zero system having unknown time-varying zeroes has been addressed. New tests have been derived, and analyzed via a simulation study. They are based upon the local approach ([5], [8], [12]) and an instrumental statistics which is "robust" with respect to the nuisance parameters.

The main conclusion is that detection and diagnosis of small (1%) changes in eigenfrequencies are possible, provided that the size of the sample is large enough (several thousands) and that there is no "masking" effect (different distances from the poles to the unit circle). This point is currently theoretically investigated.

Finally, the extension of all these tests to the vector case is possible and currently under study.

LIST OF TABLES

Table I

		1	2	3	4	5
Reference model	ρ_j	0.99 0.99	0.99 0.99	0.99 0.99 0.99		0.99 0.6
	ω_j	0.8 0.6	2.2 2.4	0.8 0.6 0.4		0.8 0.6
Changed model	ω_j	0.8 0.594	2.18 2.4	0.8 0.6 0.396	0.8 0.594 0.4	0.8 0.594
Relative magnitude/ω		1%	1%	1%	1%	1%
"Observable"		4‰	3‰	2‰	4‰	4‰

Table II

		1	2	3	4	5
H_0	global test	3.32	4.42	7.01	7.01	4.52
	sensitivity test	1.09 0.93	1.37 1.06	1.07 1.39 1.74	1.07 1.39 1.74	1.13
H_1	global test	17.99	175.69	17.51	22.10	4.12
	sensitivity test	1.24 17.90	189.32 1.18	1.96 1.14 13.50	1.74 17.43 1.68	0.90 1.01

Table III

		1	2	3
H_0	global test	1.75	4.29	4.11
	sensitivity test	0.48 0.59	0.70 1.54	0.17 0.25 0.62
H_1	global test	11.29	237.35	8.99
	sensitivity test	0.84 9.27	233.03 1.23	0.84 0.47 4.21

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