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Abstract: The problem of channel estimation in code-division multiple-access systems is considered. Using only the spreading code of the user of interest, a technique is proposed to identify the impulse response of the multipath channel from the received data sequence. While existing blind methods suffer from high computational complexity (due to large SVDs) and sensitivity to accurate knowledge of the noise subspace rank, the proposed method overcomes both problems. By employing a computationally simple matrix power that requires no a-priori knowledge of the noise subspace rank, we obtain efficient estimates of the noise subspace. The impulse response is then directly identified through a small sized (order of the channel) SVD or a least squares optimization. Both approaches (SVD and least squares) are also extended to accommodate for synchronization with respect to the user of interest. Extensive simulations demonstrate robustness of the proposed scheme and performance comparable to existing SVD techniques but at a lower computational cost.

Key-words: Channel estimation, CDMA.

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Estimation Aveugle de Canal pour le DS-CDMA

Résumé : Nous traitons le problème de l'estimation de canal pour le système CDMA (Code Division Multiple Access). En utilisant uniquement la signature de l'utilisateur d'intérêt, nous proposons une technique pour identifier la réponse impulsionelle d'un canal à trajets multiples à partir de la séquence des données reçues. Alors que les méthodes aveugles existantes sont d'une grande complexité (à cause des grandes SVD) et nécessitent une connaissance exacte du rang du sous-espace bruit, la méthode proposée surmonte ces deux problèmes. Nous employons une puissanse de matrice simple à calculer qui n'exige aucune connaissance a-priori du rang du sous-espace bruit et on obtient des estimations efficaces du sous-espace bruit. La réponse impulsionelle est alors directement identifiée par une SVD de petit taille (de l'ordre du canal) ou bien par un problème aux moindres carrés. Les deux techniques sont également généralisées pour soutenir une synchronization avec l'utilisateur d'intérêt. Des simulations multiples démontrent la robustesse de la méthode proposée et une performance comparable aux techniques SVD qui existent déjà, mais avec un coût de calcul moindre.

Mots-clés : estimation de canal, CDMA.

Contents

1 Introduction.

Code-division multiple-access (CDMA) implemented with direct-sequence (DS) spread spectrum constitutes one of the most important emerging technologies in wireless communications. It is well known that CDMA has been selected as the base for the 3-rd generation mobile telephone systems.

In a CDMA system users are capable of simultaneously transmitting in time, while occupying the same frequency band, by using a unique signature waveform assigned to each one of them. However, this important advantage does not come for free since it constitutes, at the same time, the main source of performance degradation. Indeed for every user, all other users play the role of (multiuser) interference. Several multiuser detectors have already been proposed in the literature and extensively analyzed (for details see [7]). All such detectors, in order to be practically implementable, require at least knowledge of the signature waveform of the user of interest. Assuming availability of this information, is in fact quite reasonable.

At the receiver end (the mobile unit), whenever CDMA signals propagate through a multipath environment, the effective signature signals are no longer the signature waveforms but rather the convolution of these signals with the channel impulse response. This combined waveform is also known as *composite signature*. Clearly, if we like to apply the detection structures of [7] introduced for the non-dispersive channel we need to know (or efficiently estimate) the channel impulse response. Furthermore, it is only natural to express a strong interest towards blind estimation methods, since this class of techniques does not require transmission of any training sequences.

Blind channel estimation methods for CDMA are considered in [8] and [4]. Both articles propose the recovery of the channel impulse response through a two-step procedure. The first step involves a large SVD in order to obtain a base for the noise subspace of the received signal, and the second consists in applying either a small SVD [8] or a QR-decomposition [4] of the size of the channel in order to obtain the final impulse response estimate. The employment of large SVDs (i.e. first step) in real time applications, essentially limits the use of these methods to small spreading codes. We should also mention that both approaches are very sensitive to the correct knowledge of the noise subspace rank. This parameter is not constant since it changes with the number of users accessing the CDMA channel. It turns out that even the slightest erroneous rank estimate can lead to drastic performance degradation.

An alternative approach is proposed in [6], where blind receivers are obtained through a max/min constrained optimization. Using the theoretical developments of this work, LMS and RLS blind adaptations are introduced in [9]. An extension of this work, using higher order cumulants, is reported in [10]. However, the corresponding implementation suffers from slow convergence even in the short code case while its success relies on the Gaussian noise assumption, and in particular the fact that higher order cumulants of Gaussian random variables are zero.

The idea we propose in this work overcomes all drawbacks reported above for the already existing schemes. Although our method follows the main lines of [8, 4], (involving two steps) it is characterized by an essential difference. We replace the first large SVD step by the computation of a matrix power. Despite the fact that, in theory, the power method attains the performance of SVD only in the limit (as the power tends to infinity), in practice we do not need to go beyond the third power. Furthermore this approach does not require knowledge of the noise subspace rank thus its robustness, with respect to this parameter, is guaranteed. For the second step, we may proceed either with a small sized SVD [8], or a QR decomposition [4], or finally a simple least squares (LS) approach (proposed here). As far as the latter approach is concerned, it should be mentioned that it also lends itself to the development of an efficient scheme for resolving the timing synchronization problem. Extensive simulations demonstrate rapid convergence of our method and performance which is comparable to [8, 4] but with a significantly lower computational cost. As far as the method of [6] is concerned, we should mention that the coincides with a special case our SVD version. However in [6] no convincing explanation is provided as to why this idea might be successful. We believe that with the setup introduced here this will become sufficiently clear and, furthermore, with our extended version we will be able to obtain significant performance gains.

The rest of the paper is organized as follows. In Section II, we introduce a general signal model for the multipath CDMA system. In Section III we present a version of the power method suitable for the channel estimation problem which is analyzed in Section IV. The realistic case scenario and synchronization issues are treated in Section V. Section VI contains a number of simulation comparisons between the proposed and the existing methods, and finally Section VII concludes our article.

2 Signal Model.

Consider a K-user DS-CDMA system with identical chip waveforms and signaling antipodally through a multipath channel in the presence of additive white noise (AWN), *not necessarily Gaussian*. Although CDMA systems are continuous in time, they can be adequately modeled by an equivalent discrete time system [7]. Specifically, no information is lost if we limit ourselves to the output of a chip matched filter applied to the received analog signal and sampled at the chip rate [7].

Let N be the processing gain of the code and L the length of the channel impulse response. Let $\mathbf{s}_i = [s_i(0)\, s_i(1) \cdots s_i(N-1)]^t$ be the length N normalized signature waveform of User-i (i.e. $\|\mathbf{s}_i\| = 1$), and denote by $s_i(n)$ the infinite sequence generated by zero padding the signature $\mathbf{s}_i$ from both ends towards infinity. The transmitted signal due to User-i is given by

$$z_i(n) = a_i \sum_{k=-\infty}^{\infty} s_i(n - kN - \tau_i) b_i(k), \quad i = 1, \ldots, K, \tag{1}$$

where a_i is the amplitude of User-i; $b_i(n)$ the corresponding bit sequence; and τ_i its initial delay that can take any value in the set $\{0, \ldots, N-1\}$. When $z_i(n)$ propagates through a multipath AWN channel with impulse response $\mathbf{f}_i = [f_i(0) \ \cdots \ f_i(L-1)]^t$, then the received signal $y(n)$ can be written as

$$\begin{aligned} y(n) &= \sum_{i=1}^{K} z_i(n) \star f_i(n) + \sigma w(n) \\ &= \sum_{i=1}^{K} \sum_{k=-\infty}^{\infty} a_i \tilde{s}_i(n - kN - \tau_i) b_i(k) + \sigma w(n), \end{aligned} \tag{2}$$

where $\star$ denotes convolution; $\tilde{s}_i(n) = s_i(n) \star f_i(n)$ is the convolution between the sequence $s_i(n)$ and the channel impulse response $\mathbf{f}_i$ (i.e. the composite signature of User-i zero-padded from both ends); and $w(n)$ is a unit variance i.i.d. noise sequence with σ^2 denoting the power of the AWN.

Even though the model given in (2) fully describes the uplink (mobile to base station) scenario of a multipath CDMA system, it can be used for the downlink case (base station to mobile) as well. Indeed in downlink, users are completely synchronized, therefore $\tau_1 = \cdots = \tau_K = \tau$, and they propagate through the *same* multipath channel, thus $\mathbf{f}_1 = \cdots = \mathbf{f}_K = \mathbf{f}$. It is the latter case we are going to consider here, we should however note that, with almost no modifications, our methodology can be applied for the uplink case as well, in order to estimate the different channels *one-by-one*.

Without loss of generality, throughout this article, we will assume that the user of interest is User-1. At this point we will also assume that the initial delay τ is known and therefore we have exact synchronization with the user of interest. This assumption is not restrictive since it will be relaxed in Section V. For the presentation of our method it is more convenient to express the received signal in blocks of data. In particular we are interested in blocks of size $mN + L - 1$, where m is a positive integer. Consequently let us consider the block

$$\mathbf{r}_m(n) = [y(nN) \ \cdots y((n-m)N - L + 2)]^t \tag{3}$$

which, as we said, is assumed to be synchronized with the user of interest. Notice that, due to synchronization, the block $\mathbf{r}_m(n)$ caontains m entire copies of the composite signature of the user of interest. To illustrate this fact, let us analyze the case $m=2$. Vector $\mathbf{r}_2(n)$ can then be decomposed as follows

$$\begin{aligned}\mathbf{r}_2(n) &= \begin{bmatrix}\tilde{\mathbf{s}}_1\\ \mathbf{0}_{N\times 1}\end{bmatrix} a_1 b_1(n) + \begin{bmatrix}\mathbf{0}_{N\times 1}\\ \tilde{\mathbf{s}}_1\end{bmatrix} a_1 b_1(n-1)\\ &+ \mathrm{MAI}+\mathrm{ISI}+\sigma\mathbf{w}_2(n),\end{aligned} \tag{4}$$

where $\tilde{\mathbf{s}}_1 = [\tilde{s}_1(0)\cdots\tilde{s}_1(N+L-2)]^t$ is the composite signature of User-1. We observe in (4) that the first two terms involve the entire composite signature of the user of interest; the multi-access interference (MAI) part that follows, contains similar terms coming from interfering users; then follows the ISI part that includes the inter-symbol interference of all users and finally the last term is the AWN vector. All parts in (4), except the last one, involve sums of terms of the form $\mathbf{d}_l b_i(n-j)$ where $\mathbf{d}_l$ are deterministic vectors corresponding to shifted versions of composite signatures (MAI) or shifted sections of composite signatures (ISI), and $b_i(n)$ are binary data that are mutually independent and independent from the noise vector. A final point we should make, before proceeding with the presentation of our results, is the fact that the composite signature of User-1 can always be written in a matrix product form as

$$\tilde{\mathbf{s}}_1 = \mathbf{S}_1\mathbf{f} \tag{5}$$

where $\mathbf{S}_1$ is a convolution matrix of dimensions $(N+L-1)\times L$, corresponding to the signature of User-1 and defined as

$$\mathbf{S}_1 = \begin{bmatrix} s_1(0) & 0 & \cdots & 0\\ \vdots & s_1(0) & \ddots & \vdots\\ s_1(N-1) & \vdots & \ddots & 0\\ 0 & s_1(N-1) & \ddots & s_1(0)\\ \vdots & \vdots & \ddots & \vdots\\ 0 & 0 & \cdots & s_1(N-1)\end{bmatrix}. \tag{6}$$

3 The Power Method.

Following the main lines of the method in [8, 4], our channel estimation scheme involves two main steps. The first consists in estimating a basis for the noise subspace of the received signal or, equivalently, an alternative quantity that is more suitable for the channel estimation problem. The second step, once the information regarding the noise subspace is available, consists in estimating the final channel impulse response.

Let us first consider the autocorrelation matrix $\mathbf{R}_{\mathbf{r}_m}$ of the received data vector $\mathbf{r}_m(n)$ defined in (3), then

$$\mathbf{R}_{\mathbf{r}_m} \stackrel{\triangle}{=} E\{\mathbf{r}_m(n)\mathbf{r}_m^t(n)\} = \mathbf{Q} + \sigma^2\mathbf{I} \tag{7}$$

where $\mathbf{I}$ is the identity matrix and $\mathbf{Q}$ a square matrix of dimension $(mN+L-1)$ having the form

$$\mathbf{Q} = \sum_l \mathbf{d}_l \mathbf{d}_l^t, \tag{8}$$

with $\mathbf{d}_l$ the deterministic vectors contributing to $\mathbf{r}_m(n)$ defined in (4).

Consider now the SVD of the matrix $\mathbf{Q}$

$$\mathbf{Q} = [\mathbf{U}_s \quad \mathbf{U}_n] \begin{bmatrix} \Lambda_s & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} [\mathbf{U}_s \quad \mathbf{U}_n]^t; \tag{9}$$

this leads to the following SVD of the autocorrelation matrix $\mathbf{R}_{\mathbf{r}_m}$

$$\mathbf{R}_{\mathbf{r}_m} = [\mathbf{U}_s \quad \mathbf{U}_n] \begin{bmatrix} \Lambda_s + \sigma^2\mathbf{I} & \mathbf{0} \\ \mathbf{0} & \sigma^2\mathbf{I} \end{bmatrix} [\mathbf{U}_s \quad \mathbf{U}_n]^t. \tag{10}$$

where Λ_s is a diagonal matrix containing the singular values of $\mathbf{Q}$ and $[\mathbf{U}_s\mathbf{U}_n]$ an orthonormal matrix with $\mathbf{U}_s, \mathbf{U}_n$ being orthonormal bases for the signal and noise subspace respectively. We can see that the singular values of $\mathbf{R}_{\mathbf{r}_m}$ corresponding to the noise subspace are all equal to σ^2 and are the smallest ones since the elements of Λ_s are positive. Using the decomposition in (10) we can now state the next lemma which constitutes the main idea that our method is based on.

Lemma 1 *Let $\mathbf{R}_{\mathbf{r}_m}$ be the autocorrelation matrix defined in (7) and consider the SVD in (10), then we have the following limit*

$$\lim_{k\to\infty} \left(\sigma^2\mathbf{R}_{\mathbf{r}_m}^{-1}\right)^k = \mathbf{U}_n\mathbf{U}_n^t. \tag{11}$$

Proof of Lemma 1:

Let $\lambda_1 \geq \lambda_2 \geq \ldots \geq \lambda_d > 0$ be the diagonal elements of the matrix Λ_s, then we can write

$$\left(\sigma^2\mathbf{R}_{\mathbf{r}_m}^{-1}\right)^k = [\mathbf{U}_s \quad \mathbf{U}_n] \begin{bmatrix} \left(\frac{\Lambda_s+\sigma^2\mathbf{I}}{\sigma^2}\right)^{-k} & \mathbf{0} \\ \mathbf{0} & \mathbf{I} \end{bmatrix} [\mathbf{U}_s \quad \mathbf{U}_n]^t. \tag{12}$$

From the above equality we can see that the only term in the right hand side that depends on the power k is the matrix $\left(\frac{\Lambda_s+\sigma^2\mathbf{I}}{\sigma^2}\right)^{-k}$. *Since* $\frac{\Lambda_s+\sigma^2\mathbf{I}}{\sigma^2}$ *is diagonal with elements of the form* $\frac{\lambda_i+\sigma^2}{\sigma^2} > 1$, *for* $i = 1,\ldots,d$, *we may deduce that*

$$\lim_{k\to\infty}\left(\frac{\Lambda_s+\sigma^2\mathbf{I}}{\sigma^2}\right)^{-k} = \mathbf{0}. \tag{13}$$

Finally, combining (12) and (13) we can easily verify the validity of (11). ■

Lemma 1 is a variant of the *power method* [1] used in numerical analysis for estimating the subspace corresponding to the *maximum* singular value. Since σ^2 is the smallest singular value of $\mathbf{R}_{\mathbf{r}_m}$, by considering the inverse matrix $\mathbf{R}_{\mathbf{r}_m}^{-1}$ in the lemma, the noise subspace $\mathbf{U}_n$ is mapped to $1/\sigma^2$ which becomes the largest singular value of the inverse matrix. Our intention is to use the power $\mathbf{R}_{\mathbf{r}_m}^{-k}$ for approximating the product $\mathbf{U}_n\mathbf{U}_n^t$. Since, as we can see in (13), the speed of convergence is exponential with the *slowest* component being of the form $(\sigma^2/(\lambda_d+\sigma^2))^k$, we can clearly deduce that the power method is more efficient in high SNR cases. In other words we expect in high SNR to approximate more efficiently the desired product with smaller powers; a fact that will also become apparent in our simulations.

4 Key Results.

Channel estimation methods that use SVD of the autocorrelation matrix $\mathbf{R}_{\mathbf{r}_m}$ are based on the following key idea. Since from SVD the matrix $[\mathbf{U}_s\,\mathbf{U}_n]$ is orthonormal we have that $\mathbf{U}_n^t\mathbf{U}_s = 0$. This suggests that for any vector $\mathbf{s}$ belonging to the signal subspace we also have

$$\mathbf{U}_n^t\mathbf{s} = 0. \tag{14}$$

In particular this is true for all vectors $\mathbf{d}_l$ contributing to the matrix $\mathbf{Q}$ defined in (8). We have now the following lemma which is a straightforward consequence of (14).

Lemma 2 *Let* $\mathbf{f}$ *denote the channel impulse response that we like to estimate; if* $\mathbf{F}_i$, $i = 1,\ldots,l$, *are matrices such that all vectors* $\mathbf{F}_i\mathbf{f}$ *belong to the signal subspace then*

$$(\mathbf{U}_n^t\mathbf{F})\mathbf{f} = 0, \text{ where } \mathbf{F} = \mathbf{F}_1+\cdots+\mathbf{F}_l. \tag{15}$$

From (15) it is clear that if we had available the *exact* noise subspace basis $\mathbf{U}_n$ and a matrix $\mathbf{F}$ then we could recover $\mathbf{f}$ as a vector belonging to the *null space* of the matrix $\mathbf{U}_n^t\mathbf{F}$.

4.1 Identifiability.

In this subsection, we will briefly consider the problem of consistency of the channel impulse response estimates through (15). We must stress that identifiability issues are in general complicated; for a more rigorous analysis one can consult [3]. Here we would like to adopt a rather simplistic approach that provides a better insight of the problem and also leads to necessary conditions.

Consider the autocorrelation matrix $\mathbf{R}_{\mathbf{r}_m}$ of dimensions $(mN+L-1)\times(mN+L-1)$ and the corresponding SVD of (10). Let us denote by r_s, r_n the signal and noise subspace ranks respectively, then the matrix $\mathbf{U}_n^t\mathbf{F}$ in (15) is of dimensions $r_n \times L$. If $\mathbf{U}_n$ is the exact noise subspace and $\mathbf{F}$ is a matrix such that $\mathbf{Ff}$ belongs to the signal subspace then we can conclude that, due to (15), the column rank of $\mathbf{U}_n^t\mathbf{F}$ can, *at most*, be equal to $L-1$. In order for (15) to have a unique solution (modulo a multiplicative constant-ambiguity) the column rank of $\mathbf{U}_n^t\mathbf{F}$ must be *exactly equal* to $L-1$. Since the column rank of a matrix is equal to its row rank (and also equal to the rank of the matrix) we conclude that in order to have a row rank equal to $L-1$ a *necessary* condition is to have at least $L-1$ rows, that is, $r_n \geq L-1$. Since $r_s + r_n = mN + L - 1$ this yields the following necessary condition

$$r_s \leq mN. \tag{16}$$

Let us now specify, more precisely, the signal subspace rank. Notice that the number of columns of $\mathbf{U}_s$ is equal to r_s. In fact $\mathbf{U}_s$ is an orthonormal basis for the subspace spanned by the vectors $\mathbf{d}_l$ introduced in (8). For the sake of clarity we present these vectors in Fig. 1 for the downlink scenario. We recall that in this case all K users are synchronized. As we can see there are m big rectangles of dimensions $(N+L-1)\times K$, containing entire composite signatures of all K users. The first such rectangle corresponds to the n-th user-bits whereas the last to the $(n-m+1)$-st. The two small rectangles, with dimensions $(L-1)\times K$, contain ISI coming from the $(n+1)$-st and $(n-m)$-th user-bits respectively. Each rectangle has a rank that cannot exceed its *smallest* dimension. Assuming that the number of users K is smaller than the processing gain N we conclude that

$$r_s \leq mK + 2\min\{L-1, K\}. \tag{17}$$

We therefore deduce that if we select m such that $mK + 2\min\{L-1,K\} \leq mN$ then the validity of the necessary condition (16) is guaranteed. This yields the following estimate for the number of blocks m

$$m \geq \frac{2\min\{L-1,K\}}{N-K}. \tag{18}$$

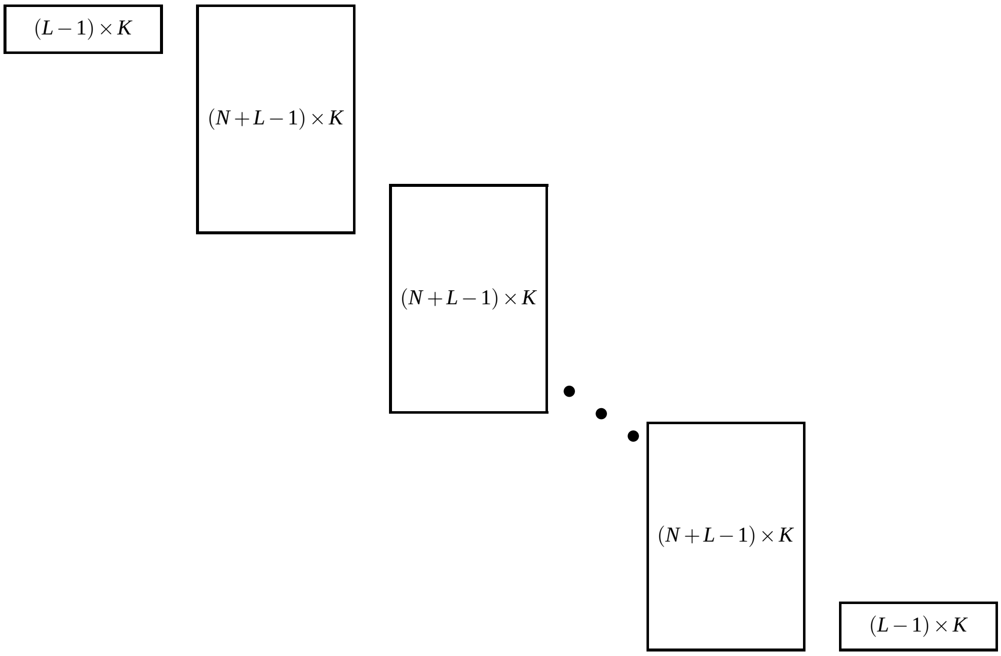

Figure 1: Representation of the vectors composing the signal subspace.

Equivalently, for a given number of blocks m, we can obtain an upper bound for the maximum load of the system

$$K \leq N - 2\min\left\{\frac{N}{m+2}, \frac{L-1}{m}\right\}. \tag{19}$$

If we like to follow the same analysis for the uplink scenario then, due to lack of synchronization, relation (17) becomes $r_s \leq (m+2)K$, yielding

$$m \geq \frac{2K}{N-K} \text{ or } K \leq \frac{m}{m+2}N \tag{20}$$

as a possible estimate for m (for given K) or an upper bound for K (for given m). As was stated before, a more rigorous analysis can be found in [3], addressing also the identifiability problem in the case where for L we know only an upper bound. We must stress that the bounds introduced in (18) and (19) are by no means strict and must therefore be used with caution. We recall that they simply ensure validity of the *necessary condition* (16) and are thus not sufficient for identifiability. In numerous simulations, however, they turned out to be very accurate. Unfortunately we were not able to prove their sufficiency.

4.2 Two Optimization Problems.

Let us now see how we can use Equ. (15) in combination with the power method introduced in Section III in order to solve the channel estimation problem. We first observe that any $\mathbf{f}$ satisfying (15) also satisfies the following equation

$$\mathbf{f}^t\mathbf{F}^t\mathbf{U}_n\mathbf{U}_n^t\mathbf{F}\mathbf{f} = 0. \tag{21}$$

Furthermore, if for any vector $\mathbf{h}$ of length L, we define the nonnegative measure

$$V(\mathbf{h}) = \mathbf{h}^t\mathbf{F}^t\mathbf{U}_n\mathbf{U}_n^t\mathbf{F}\mathbf{h}, \tag{22}$$

then Equ. (21) suggests the following two optimization problems for recovering the channel impulse response $\mathbf{f}$.

The first method consists in solving the minimization problem

$$\hat{\mathbf{f}} = \arg\min_{\mathbf{h}} V(\mathbf{h});\ \text{subject to } \|\mathbf{h}\| = 1. \tag{23}$$

Thus $\mathbf{f}$ is being estimated as the singular vector corresponding to the smallest singular value of the matrix $\mathbf{F}^t\mathbf{U}_n^t\mathbf{U}_n\mathbf{F}$. We should note that the SVD problem suggested by (23) is only of size L (channel length).

The second method relies on a constrained least squares (LS) minimization. More specifically let

$$\mathbf{h}_i = \arg\min_{\mathbf{h}} V(\mathbf{h});\ \text{subject to } \mathbf{h}^t\mathbf{e}_i = 1, \tag{24}$$

where $\mathbf{e}_i,\ i = 1,\ldots,M$, are pre-specified vectors. The final candidate vector is the one that is closest to the SVD solution, that is, $\hat{\mathbf{f}} = \mathbf{h}_{i_o}/\|\mathbf{h}_{i_o}\|$, where i_o is given by

$$i_o = \arg\min_{i} V(\mathbf{h}_i)/\|\mathbf{h}_i\|^2. \tag{25}$$

In the ideal case when we have exact knowledge of the noise subspace, due to (21), the optimum value $V(\hat{\mathbf{f}})$ of the criterion $V(\mathbf{h})$ in both, SVD and LS approaches, becomes exactly zero. Moreover, for LS a single vector $\mathbf{e}_i$ would suffice to exactly determine $\mathbf{f}$. When however only estimates of the noise subspace are available, then the minimum value of the criterion is positive and for LS we need to use more than one vectors $\mathbf{e}_i$ in order to efficiently approximate the performance of the SVD solution (23). Finally we should note that the estimates $\hat{\mathbf{f}}$ that we obtain with both methods differ from the true $\mathbf{f}$ by a multiplicative constant (multiplicative constant ambiguity). In fact if we assume (without loss of generality) that the norm of $\mathbf{f}$ is unity, then in the estimate $\hat{\mathbf{f}}$ we have only a sign ambiguity.

Notice now that in both optimizations (23) and (24), it is the product $\mathbf{U}_n\mathbf{U}_n^t$ that appears as part of the function $V(\mathbf{h})$. As we have seen in Lemma 1 this quantity can be approximated, to any degree, by the corresponding matrix power

$$\widehat{\mathbf{U}_n\mathbf{U}_n} = \sigma^{2k}\mathbf{R}_{\mathbf{r}_m}^{-k} \tag{26}$$

yielding the following approximation for the measure $V(\mathbf{h})$ defined in (22)

$$\hat{V}(\mathbf{h}) = \sigma^{2k}\mathbf{h}^t\mathbf{F}^t\mathbf{R}_{\mathbf{r}_m}^{-k}\mathbf{F}\mathbf{h}. \tag{27}$$

Using $\hat{V}(\mathbf{h})$, the two optimization problems become

$$\hat{\mathbf{f}} = \arg\min_{\mathbf{h}} \hat{V}(\mathbf{h}) = \arg\min_{\mathbf{h}} \mathbf{h}^t\mathbf{F}^t\mathbf{R}_{\mathbf{r}_m}^{-k}\mathbf{F}\mathbf{h}; \text{ subject to } \|\mathbf{h}\| = 1, \tag{28}$$

corresponding to the SVD version of (23); and

$$\mathbf{h}_i = \arg\min_{\mathbf{h}} \hat{V}(\mathbf{h}) = \arg\min_{\mathbf{h}} \mathbf{h}^t\mathbf{F}^t\mathbf{R}_{\mathbf{r}_m}^{-k}\mathbf{F}\mathbf{h}; \text{ subject to } \mathbf{h}^t\mathbf{e}_i = 1, \tag{29}$$

with $\hat{\mathbf{f}} = \mathbf{h}_{i_o}/\|\mathbf{h}_{i_o}\|$ and i_o satisfying

$$i_o = \arg\min_i \hat{V}(\mathbf{h}_i)/\|\mathbf{h}_i\|^2 = \arg\min_i \mathbf{h}_i^t\mathbf{F}^t\mathbf{R}_{\mathbf{r}_m}^{-k}\mathbf{F}\mathbf{h}_i/\|\mathbf{h}_i\|^2, \tag{30}$$

for the LS version of (24). If in particular in the LS problem we select $\mathbf{e}_i = [0\cdots 0\,1\,0\cdots 0]^t$, for $i = 1,\ldots,L$, with the unity being in the i-th position then (29) yields

$$\mathbf{h}_i = \frac{(\mathbf{F}^t\mathbf{R}_{\mathbf{r}_m}^{-k}\mathbf{F})^{-1}\mathbf{e}_i}{\mathbf{e}_i^t(\mathbf{F}^t\mathbf{R}_{\mathbf{r}_m}^{-k}\mathbf{F})^{-1}\mathbf{e}_i}, \tag{31}$$

and the final channel estimate becomes $\hat{\mathbf{f}} = \mathbf{h}_{i_o}/\|\mathbf{h}_{i_o}\|$, where

$$\begin{aligned} i_o &= \arg\min_i \frac{1}{\mathbf{e}_i^t(\mathbf{F}^t\mathbf{R}_{\mathbf{r}_m}^{-k}\mathbf{F})^{-1}\mathbf{e}_i} \\ &= \arg\max_i \mathbf{e}_i^t(\mathbf{F}^t\mathbf{R}_{\mathbf{r}_m}^{-k}\mathbf{F})^{-1}\mathbf{e}_i \end{aligned} \tag{32}$$

It is more convenient to view the solution of the LS problem as first computing the inverse matrix $(\mathbf{F}^t\mathbf{R}_{\mathbf{r}_m}^{-k}\mathbf{F})^{-1}$, then selecting its largest diagonal element whose position identifies i_o; the final channel estimate is then the i_o-th row (or column) of $(\mathbf{F}^t\mathbf{R}_{\mathbf{r}_m}^{-k}\mathbf{F})^{-1}$ normalized to unit norm.

Notice that although for the approximation of the product $\mathbf{U}_n\mathbf{U}_n^t$ through Lemma 1 we need σ^2, this quantity plays absolutely no role in the estimation of $\mathbf{f}$ through (28) or (29). Another interesting point is the fact that for this approximation *no knowledge of the noise subspace rank is necessary*. This should be compared to the large SVD applied to $\mathbf{R}_{\mathbf{r}_m}$ in [8, 4] where for the determination of $\mathbf{U}_n$, the knowledge of this parameter (or a reliable estimate) is indispensable.

5 Channel Estimation Method and Synchronization.

In this section we probe further into our problem by exploiting the special structure of the $\mathbf{d}_l$-vectors introduced in (8) that correspond to the user of interest. For the application of the two estimation schemes (SVD and LS) presented in the previous section, we need to have a *known* matrix $\mathbf{F}$ such that $\mathbf{Ff}$ belongs to the signal subspace. It turns out that such a matrix is easy to obtain. For illustration let us again consider the case $m=2$. From (4) we have that the two vectors $[\tilde{\mathbf{s}}_1^t \ \mathbf{0}_{1\times N}]^t$ and $[\mathbf{0}_{1\times N} \ \tilde{\mathbf{s}}_1^t]^t$ belong to the signal subspace and, using (5), they can be written under the form $\mathbf{F}_i\mathbf{f}, \ i=1,2$, where

$$\mathbf{F}_1 = \begin{bmatrix} \mathbf{S}_1 \\ \mathbf{0}_{N\times L} \end{bmatrix}, \ \ \mathbf{F}_2 = \begin{bmatrix} \mathbf{0}_{N\times L} \\ \mathbf{S}_1 \end{bmatrix}. \tag{33}$$

Since we assume that the signature of the user of interest is known, we conclude that both matrices $\mathbf{F}_i, \ i=1,2$ are known as well and so is their sum $\mathbf{F}=\mathbf{F}_1+\mathbf{F}_2$. This property can be generalized to the m-block case in a straightforward manner and we can write

$$\mathbf{F} = \sum_{l=0}^{m-1} \begin{bmatrix} \mathbf{0}_{lN\times L} \\ \mathbf{S}_1 \\ \mathbf{0}_{(m-1-l)N\times L} \end{bmatrix}. \tag{34}$$

Matrix $\mathbf{F}$ turns out to have a simple structure. In particular, it is a convolution matrix as in (6), but of dimensions $(mN+L-1)\times L$, where the first column contains the signature $\mathbf{s}_1$ *repeated m* times, i.e. it is of the form $[\overbrace{\mathbf{s}_1^t \ \cdots \ \mathbf{s}_1^t}^{m \text{ times}} \ \mathbf{0}_{1\times L-1}]^t$.

Since we have available a known $\mathbf{F}$ matrix we can now proceed to the computation of the autocorrelation matrix. We can approximate $\mathbf{R}_{\mathbf{r}_m}$ by using the sample autocorrelation matrix of the received data vector, namely

$$\hat{\mathbf{R}}_{\mathbf{r}_m} = \frac{1}{T}\sum_{n=1}^{T} \mathbf{r}_m(n)\mathbf{r}_m^t(n). \tag{35}$$

Thus we have now available all necessary parameters in order to solve the two optimization problems in (28) and (32).

Remark: A subtle and very important remark regarding Lemma 1 concerns the employment of possible values for the power k. Notice that the limit in (11) is correct, i.e. we obtain the product $\mathbf{U}_n\mathbf{U}_n^t$, *only* when the singular values corresponding to the noise subspace are *exactly* equal. Unfortunately, with the approximation in (35) this is rarely the case. This observation leads to a twofold deduction. First, considering that the number of bits T is constant, the employment of continuously increasing powers k does not necessarily lead to improved performance. This is because the corresponding limit (11) instead of being the desired product will become just the rank-one matrix $\mathbf{u}\mathbf{u}^t$ where $\mathbf{u}$ is the singular vector corresponding to the smallest singular value of $\hat{\mathbf{R}}_{\mathbf{r}_m}$. On the other hand, for any two constant powers, as the number of bits T grows, eventually the larger power will prevail. In the simulations section we will have the opportunity to confirm these observations.

Our proposed versions are clearly blind since they require only the received data sequence $\mathbf{r}_m(n)$ and knowledge of the signature $\mathbf{s}_1$ of the user of interest. We should also note that computing the product $\mathbf{U}_n\mathbf{U}_n^t$ with the power method *can* be computationally very efficient and even lead to on-line adaptive implementations. Although this is not a subject that we would like to raise here, since it is still under investigation, we can easily see that by adapting the inverse of the sample autocorrelation matrix $\hat{\mathbf{R}}_{\mathbf{r}_m}$ using RLS, we immediately gain an order of magnitude in computations as compared to applying directly SVD at each step. This is true because, as we shall see in the simulations, in order for our versions to match the performance of SVD we do not need to use powers beyond $k = 3$.

As was previously indicated, in [8, 4], where the matrix $\mathbf{U}_n$ is estimated instead, there is also the need to reliably identify the rank of the noise subspace. Such estimates are performed with the help of Akaike's information criterion or the minimum description length criterion (for details see [8]). Unfortunately, it turns out that, the schemes in [8, 4] tend to be very sensitive with respect to the exact knowledge of the noise subspace rank. In particular, they can exhibit considerable performance degradation even when the corresponding estimate is slightly erroneous, as we will have the chance to find out in the next section. Finally we should note that the method proposed in [6] coincides with our SVD version with the special selection $k = 1$. Although this specific choice provides efficient channel estimates in high SNR, for medium to low SNR cases, significant performance gains can be obtained by employing higher powers.

5.1 Synchronization.

Up to this point we assumed that the receiver, more precisely the data blocks $\mathbf{r}_m(n)$, were synchronized with the user of interest. Now we are going to relax this assumption and propose a simple and efficient means for synchronization. We recall that the initial delay τ of User-1 can take any value in the set $\{0,\ldots,N-1\}$.

Following the idea introduced in [4], we observe that when the second order statistics are available (ideal case) and the block $\mathbf{r}_m(n)$ is synchronized with the user of interested then, in both approaches (SVD and LS), the *optimum value* of the measure $V(\mathbf{h})$ is equal to zero. When on the other hand the data block is not synchronized, the *optimum value* of the measure cannot attain this lower limit since Equ. (15) is no longer valid. In other words the optimum value of $V(\mathbf{h})$, as a function of the data block timing, *is minimized* when the data block $\mathbf{r}_m(n)$ is synchronized with the user of interest. This is exactly the property we are going to use to perform synchronization. In other words we propose to solve either the SVD or the LS minimization problem for all N possible timing values of the data block. We can then select as the estimate of the initial delay the one with the smallest optimum $\hat{V}(\mathbf{h})$ value.

A notable characteristic of the N minimizations involved in the synchronization problem is the fact that the (sample) autocorrelation matrices for consecutive delay values *have a significant overlap*. By exploiting this property we will now present an efficient method for computing certain matrices entering in the two optimization problems. Let us denote by $\mathbf{r}_m(n,\tau)$ the following vector of length $mN+L-1$

$$\mathbf{r}_m(n,\tau)=[y(nN-\tau)\ \cdots y((n-m)N-L+2-\tau)]^t, \tag{36}$$

where we have introduced a delay parameter τ. We can now, similarly to (35), define the corresponding sample autocorrelation matrix as follows

$$\hat{\mathbf{R}}_\tau=\frac{1}{T}\sum_{n=1}^{T}\mathbf{r}_m(n,\tau)\mathbf{r}_m^t(n,\tau), \tag{37}$$

where for the sake of simplicity, we have not included the index $\mathbf{r}_m$ in the definition in (37). One may easily observe that between the two matrices $\hat{\mathbf{R}}_\tau,\hat{\mathbf{R}}_{\tau+1}$ for any $0\le\tau\le N-2$ there is a square overlap of dimensions $mN+L-2$. This is shown more clearly from the following relation

$$\mathcal{D}_\tau=\begin{bmatrix}\hat{\mathbf{R}}_\tau & \mathbf{a}_\tau\\ \mathbf{a}_\tau^t & b_{\tau+mN+L}\end{bmatrix}=\begin{bmatrix}b_\tau & \mathbf{b}_\tau^t\\ \mathbf{b}_\tau & \hat{\mathbf{R}}_{\tau+1}\end{bmatrix}, \tag{38}$$

where

$$b_\nu = \frac{1}{T}\sum_{n=1}^{T} y^2(nN-\nu), \tag{39}$$

$$\mathbf{a}_\tau = \frac{1}{T}\sum_{n=1}^{T} \mathbf{r}_m(n,\tau)y((n-m)N-L+1-\tau), \tag{40}$$

$$\mathbf{b}_\tau = \frac{1}{T}\sum_{n=1}^{T} \mathbf{r}_m(n,\tau+1)y(nN-\tau). \tag{41}$$

Using the Schur inverse complement [2] one can compute the matrix $\hat{\mathbf{R}}_{\tau+1}^{-1}$, from $\hat{\mathbf{R}}_\tau^{-1}$ with a rank-two modification. We first compute $\mathcal{D}_\tau^{-1}$ as follows

$$\mathcal{D}_\tau^{-1} = \begin{bmatrix} \hat{\mathbf{R}}_\tau^{-1} & \mathbf{0} \\ \mathbf{0}^t & 0 \end{bmatrix} + \frac{1}{p_1}\begin{bmatrix} \mathbf{y} \\ 1 \end{bmatrix}\begin{bmatrix} \mathbf{y}^t & 1 \end{bmatrix}, \tag{42}$$

where

$$p_1 = b_{\tau+mN+L} - \mathbf{a}_\tau^t \hat{\mathbf{R}}_\tau^{-1}\mathbf{a}_\tau, \tag{43}$$

$$\mathbf{y} = -\hat{\mathbf{R}}_\tau^{-1}\mathbf{a}_\tau. \tag{44}$$

Then, based on (38), we can obtain $\hat{\mathbf{R}}_{\tau+1}^{-1}$ from $\mathcal{D}_\tau^{-1}$ from the relation

$$\begin{bmatrix} 0 & \mathbf{0}^t \\ \mathbf{0} & \hat{\mathbf{R}}_{\tau+1}^{-1} \end{bmatrix} = \mathcal{D}_\tau^{-1} - \frac{1}{p_2}\begin{bmatrix} 1 \\ \mathbf{x} \end{bmatrix}\begin{bmatrix} 1 & \mathbf{x}^t \end{bmatrix}, \tag{45}$$

where

$$p_2 = \frac{b_\tau^2}{b_\tau + \begin{bmatrix} 0 & \mathbf{b}_\tau^t \end{bmatrix}\mathcal{D}_\tau^{-1}\begin{bmatrix} 0 \\ \mathbf{b}_\tau \end{bmatrix}}, \tag{46}$$

$$\begin{bmatrix} 0 \\ \mathbf{x} \end{bmatrix} = -\left(\frac{p_2}{b_\tau}\right)\mathcal{D}_\tau^{-1}\begin{bmatrix} 0 \\ \mathbf{b}_\tau \end{bmatrix}. \tag{47}$$

Computing $\hat{\mathbf{R}}_{\tau+1}^{-1}$ from the $\hat{\mathbf{R}}_\tau^{-1}$ requires $O((mN+L-1)^2)$ operations while computing the inverse directly has complexity $O((mN+L-1)^3)$; thus gaining an order of magnitude in operations.

6 Simulations - Discussion.

In this section, we perform several simulations to demonstrate the satisfactory performance of the blind channel estimation schemes developed in the previous section. We focus on the downlink scenario, and consider various cases according to the size of the processing gain. More specifically, we examine the performance of our algorithms for signature waveforms of length $N = 16$ and $N = 128$ under diverse signaling conditions. Since the methods developed in [8, 4] present very similar performances we will compare our schemes only to [8]. Furthermore, regarding the estimation scheme of [6], we recall that it coincides with our SVD, $k = 1$ version. We first proceed with spreading codes of $N = 16$ and then continue to the $N = 128$ case.

6.1 Spreading Codes of Length 16.

Randomly generated sequences of length $N = 16$ are used as spreading codes. Once generated, the codes are kept constant for the whole simulation set. We use the length $L = 3$ channel of [5] (known to be a "difficult case" due to the deep null in its frequency response). The number of blocks that are processed together is $m = 3$. This selection, according to (19), allows the load of the system to go up to 14 users. For our simulations we select $K = 10$ users. User-1 is assumed to be the user of interest having unit power. All other users are assumed to have the same power level, which is 20 db higher than User-1.

We perform our first simulation with the conditions previously stated and with SNR = 10 db. Fig. 2 depicts the channel estimation error power, averaged over 100 independent runs, as a function of the number of bits. All graphs start at bit 50, in order for the matrix $\hat{\mathbf{R}}_{\mathbf{r}_3}$ to become of full rank. We observe that the SVD version outperforms the LS for the same value of the power k. Undoubtedly, we can also remark the excellent performance of the SVD even for power $k = 1$, attaining very rapidly an estimation error that lies more than 15 db below the noise level. The performance of both versions with $k = 3$ matches the computationally demanding method of [8], which is clearly superior to the performance of the $k = 1$ case.

For the second simulation we consider an SNR of 20 db, only here we also examine the performance of the method in [8] when there is an underestimation of the signal subspace rank just by a single unity. Fig. 3 depicts the performance of the competing schemes. The method of [8] with incorrect rank estimate corresponds to the dashed line whereas the one with the correct rank to the solid line. We observe the considerable performance degradation (of approximately 25 db) when we have a slightly incorrect rank estimate. On the other hand both proposed SVD and LS versions are insensitive to this parameter. In this high

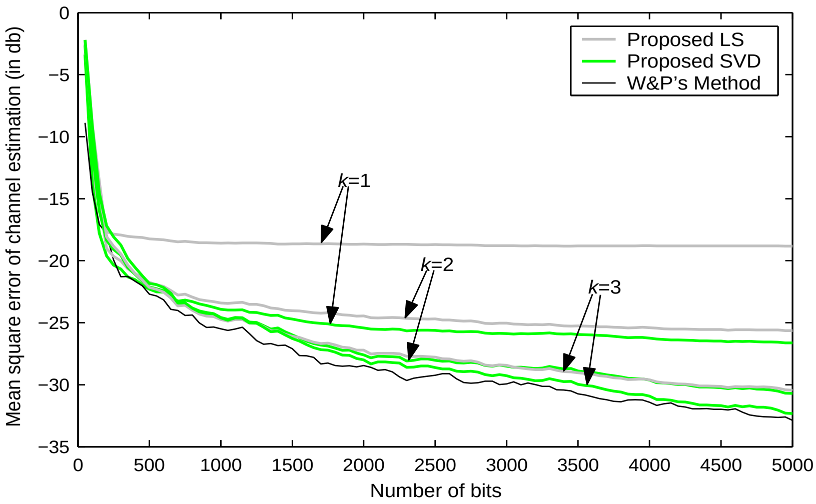

Figure 2: Performance of proposed channel estimation schemes versus the method of [8]; codes of length $N = 16$; noise power 10 db.

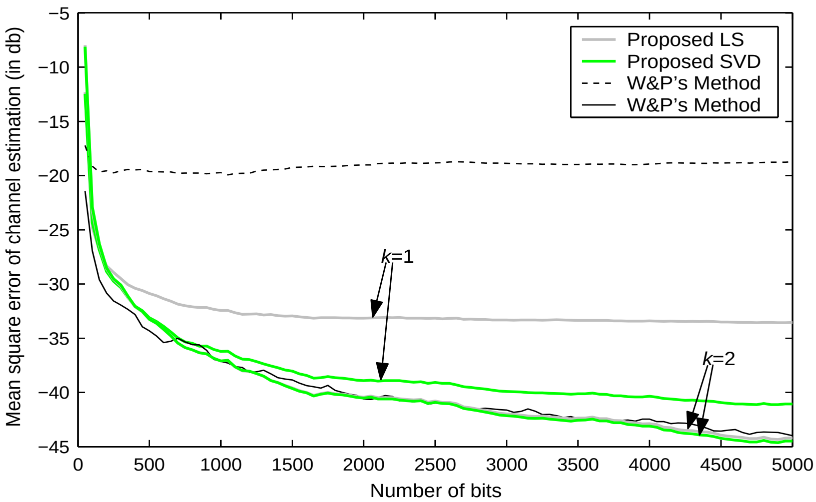

Figure 3: Performance of the method of [8] with (solid) and without (dashed) correct subspace rank estimation versus the proposed schemes; codes of length $N = 16$; noise power 20 db.

SNR environment, we can also see that their performance is maximized with $k = 2$. Again, all graphs are the result of an average of 100 independent runs.

In the third simulation we examine the performance of the synchronization method proposed in the previous section. In Fig. 4 we plot the optimum value of $\hat{V}(\mathbf{h})$, defined in (27), as a function of the timing parameter τ and the power k. The exact initial delay is set to $\tau = 5$. To facilitate comparisons, each graph is normalized so that its maximum value is equal to unity. In this simulation the users are under perfect power control and the

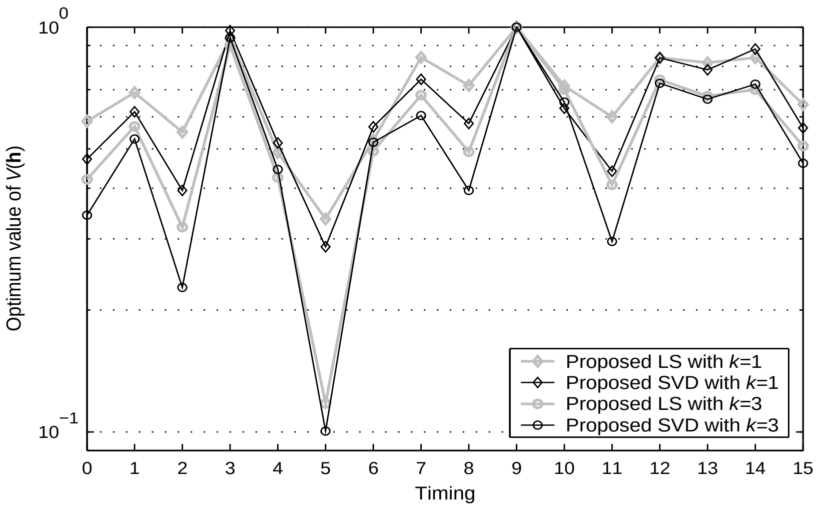

Figure 4: Initial delay estimation using the proposed SVD and LS schemes; codes of length $N = 16$; noise power 10 db.

SNR is 10 db. The number of bits used to obtain the results presented in Fig. 4 is 200. We can see that both versions achieve exact estimation of the initial delay and have comparable performance for the same value of the power k. Moreover, with $k = 3$ the correct timing is more clearly observed in both methods as compared to the case $k = 1$. We should also mention that in 10000 independent runs both versions *always* identified correctly the value of the delay, with all powers $k = 1, 2, 3$.

6.2 Spreading Codes of Length 128.

In the present subsection we simulate the proposed LS and SVD schemes with signature waveforms of a much larger size. We consider a DS-CDMA system with processing gain equal to $N = 128$. The channel vector consists of $L = 10$ coefficients (also taken from [5]) and the number of blocks processed together is, as before, $m = 3$. The load upper bound from (19) becomes 121 users. The total number of users considered here is $K = 80$, with 29 of them having the same power as User-1; 30 users being 10 db stronger; and the remaining 20 being 20 db stronger than the user of interest. Again all graphs are the average of 100 independent runs.

The performance of the LS and SVD version is presented in Fig. 5 for an SNR level of 10 db. Due to the use of large signature waveforms the graphs start from bit 400 in order for the initial autocorrelation matrix to be of full rank. The method of [8] is computed every 500 bits due to its high computational complexity. Again we can observe that the SVD version

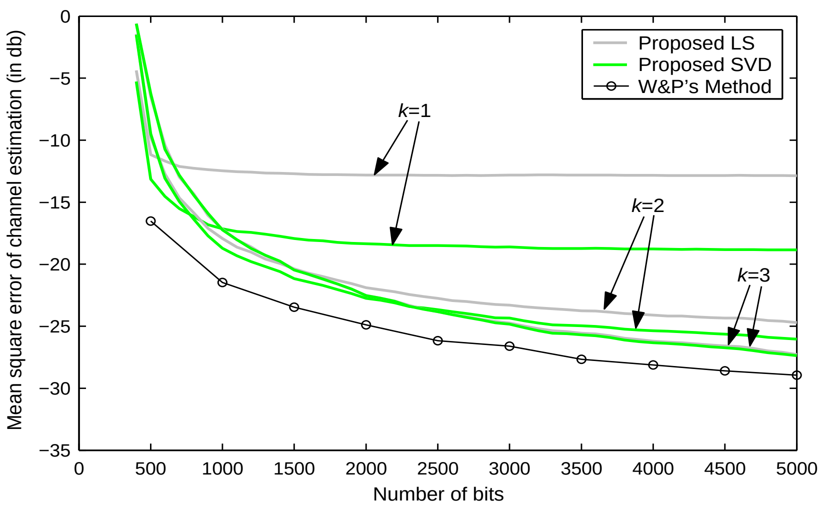

Figure 5: Performance of proposed channel estimation schemes versus the method of [8]; codes of length $N = 128$; noise power 10 db.

outperforms the LS for the same value of power k. Moreover, for $k = 3$ the performance of both methods is maximized, resulting quickly in a mean square error of 28 db, and becoming practically the same. The method of [8] attains a performance which differs from our $k = 3$ case by less than 1.5 db. We can also observe the significant performance gains (more than an order of magnitude) obtained by employing powers higher than $k = 1$.

Finally, in the last example depicted in Fig. 6 the same scenario is considered, i.e. all parameters remain the same, except the SNR which becomes 20 db. We can see that the SVD approach exhibits an excellent performance even from the first power $k = 1$, while both methods attain maximum (in fact indistinguishable) performance for $k = 2$. In this high

SNR environment our two proposed versions with $k = 2$ exhibit less than 1 db difference from [8].

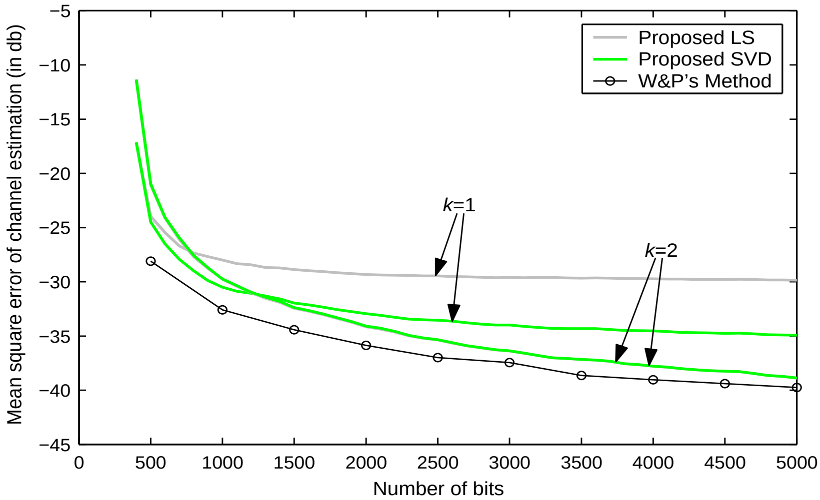

Figure 6: Performance of proposed channel estimation schemes versus the method of [8]; codes of length $N = 128$; noise power 20 db.

It is clear from our simulations that in the majority of cases and even under extreme signaling conditions, it suffices to consider a power of $k = 3$. As we pointed out earlier, what is also interesting regarding the proposed methodology, is the fact that it lends itself to the development of efficient adaptive schemes using RLS (and LMS) type recursions. Adaptive realizations for the method of [8] are also possible through subspace tracking algorithms however such approaches are even more sensitive to the knowledge of the subspace rank than off line techniques.

7 Conclusion.

In this work we examined the blind channel estimation problem for DS-CDMA in multipath AWN channels and considered a similar to [8, 4] two-step methodology for its solution. The novelty of our method consists in replacing the first, computationally demanding step of [8, 4], involving a large SVD, with a simple and computationally efficient matrix power. As far as the second step of our method is concerned, except the small sized SVD proposed in [8], or the QR decomposition of [4], we also introduced a least squares scheme. The two versions (LS and SVD) were tested under diverse signaling conditions and always compared very favorably to the methods of [8, 4] but at a significantly lower computational cost. Unlike however the latter methods, our approach does not require any a-priori knowledge or estimates of the signal subspace rank, since it is completely independent of this parameter. As far as the approach in [6] is concerned, we should mention that it coincides with our SVD version with $k = 1$. Here however, by employing higher powers of k, we can obtain superior performance especially in medium to low SNR environments.

Finally, both proposed versions were also extended in order to allow for synchronization with the user of interest. In particular by exploiting the special structural characteristics of the synchronization problem, we developed an efficient scheme that reduces the complexity of the most computationally demanding part, by an order of magnitude.

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