Decision Directed Algorithms
for
Multiuser Detection

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Outline of the presentation

- The multiuser detection problem
- Existing detection schemes
- Linear MMSE detectors
- Linear constraint MMSE detectors
- Performance analysis of trained and decision directed linear constraint MMSE detectors
- Simulations
- Conclusion

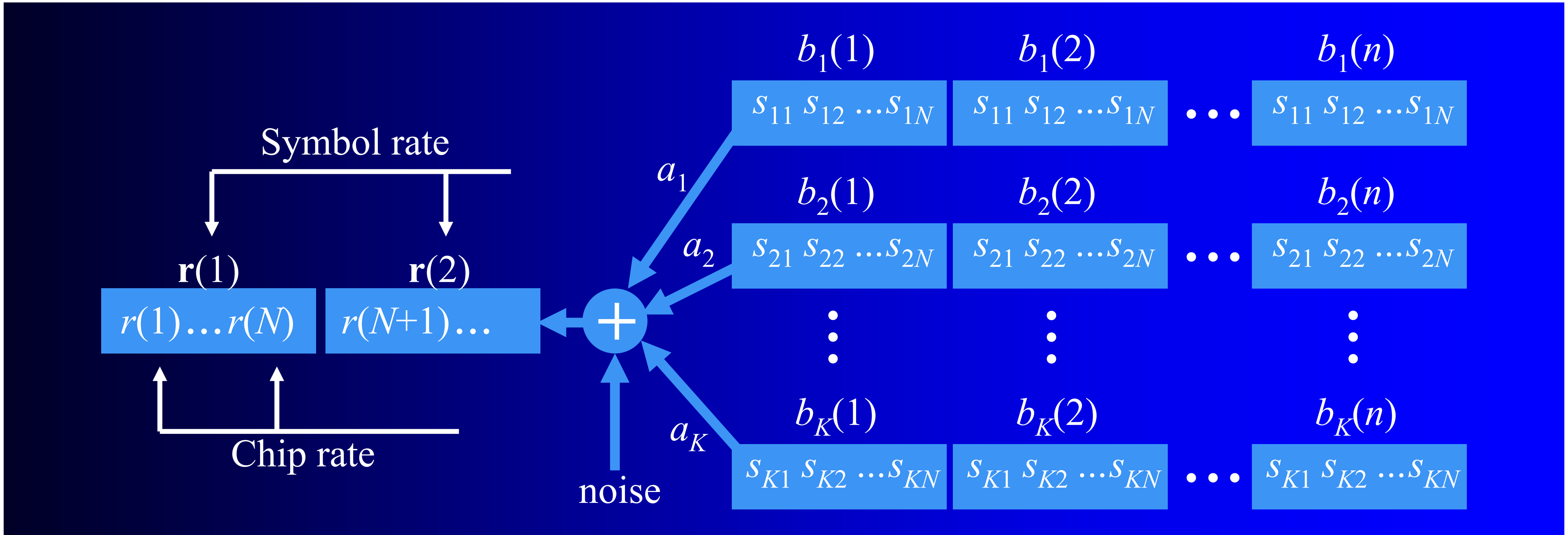
The multiuser detection problem

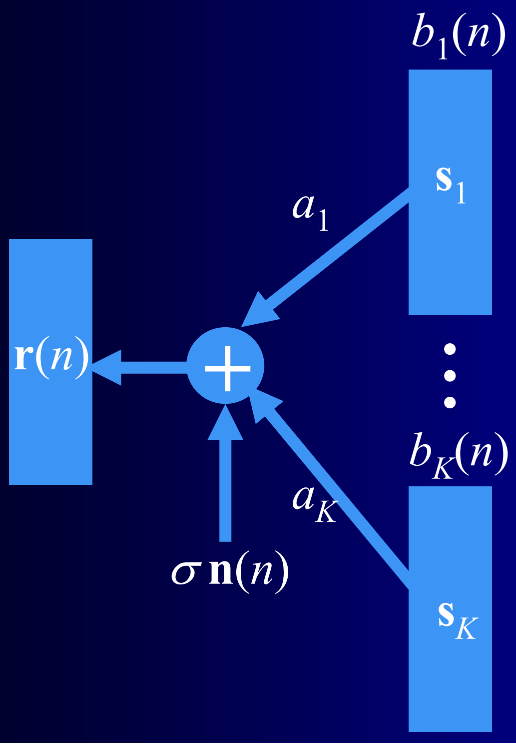
To each user it is assigned a signature signal. Let for user *i* denote its signature as

$$\mathbf{s}_i=[s_{i1}\ s_{i2}\ldots\ s_{iN}]^t$$

User-*i*, to send a “1” he transmits $\mathbf{s}_i$, whereas to send a “0” he transmits $-\mathbf{s}_i$.

For synchronous communication we can describe the received signal as follows





$$\mathbf{r}(n) = \mathbf{s}_1 a_1 b_1(n) + \mathbf{s}_2 a_2 b_2(n) + \cdots + \mathbf{s}_K a_K b_K(n) + \sigma\,\mathbf{n}(n)$$

Nonorthogonal Signatures

The MUD Problem: Assume signature $\mathbf{s}_1$ of User-1 is known and data vectors $\mathbf{r}(n)$ are available sequentially. **Estimate $b_1(n)$.**

Existing detection schemes

- **Optimum Detector (min BER)**

  Verdu (1983)

  Expensive, $\mathbf{s}_i$, a_i, σ

- **Linear Detectors**

$$\hat{b}_1(n) = \operatorname{sgn}\left(\mathbf{c}^t \mathbf{r}(n)\right)$$

Low complexity, optimum under alternative criteria.

Matched Filter Detector

$$\hat{b}_1(n) = \operatorname{sgn}\left(\mathbf{s}_1^t \mathbf{r}(n)\right)$$

Near-Far Problem

No Near-Far Problem

Reduced Performance, $\mathbf{s}_i$

Decorrelating Detector

$$\hat{b}_1(n) = \operatorname{sgn}\left(e_1^t \mathbf{R}^{-1} \mathbf{S}^t \mathbf{r}(n)\right), \quad \mathbf{S} = [\mathbf{s}_1 \; \mathbf{s}_2 \cdots \mathbf{s}_K], \quad \mathbf{R} = \mathbf{S}^t \mathbf{S}$$

$$\mathbf{r}(n) = \mathbf{s}_1 a_1 b_1(n) + \mathbf{s}_2 a_2 b_2(n) + \cdots + \mathbf{s}_K a_K b_K(n) + \sigma\, \mathbf{n}(n)$$

Linear MMSE detectors

$$\min_{\mathbf{c}} E\left\{\left(b_1(n) - \mathbf{c}^t\mathbf{r}(n)\right)^2\right\},$$ Xie et. al. (1990)

$$\mathbf{c}_o = a_1\mathbf{R}_\mathbf{r}^{-1}\mathbf{s}_1,$$

$$\mathbf{R}_\mathbf{r} = E\left\{\mathbf{r}(n)\mathbf{r}^t(n)\right\} = \mathbf{SA}^2\mathbf{S} + \sigma^2\mathbf{I},$$

$$\mathbf{A} = \mathrm{diag}\left\{a_1, \ldots a_K\right\}$$

Low complexity, optimum under alternative criteria (Near-far Resistance, Efficiency)

A-priori info: $\mathbf{s}_i$, a_i, σ

Adaptive realization (LMS):

Requires training

$$\varepsilon(n) = b_1(n) - \mathbf{c}^t(n-1)\mathbf{r}(n)$$

$$\mathbf{c}(n) = \mathbf{c}(n-1) + \mu\varepsilon(n)\mathbf{r}(n),\ \mathbf{c}(0) = \mathbf{s}_1$$

$$b_1(n) \leftarrow \hat{b}_1(n) = \mathrm{sgn}\left(\mathbf{c}^t(n-1)\mathbf{r}(n)\right)$$ (decision directed)

Minimum Output Power (MOP) (Honig,et. al, 1995)

$$\min_{\mathbf{c}} E\left\{\left(\mathbf{c}^t\mathbf{r}(n)\right)^2\right\},\ \mathbf{c}^t\mathbf{s}_1 = 1$$

$$\mathbf{c}_o = \frac{\mathbf{R}_\mathbf{r}^{-1}\mathbf{s}_1}{\mathbf{s}_1^t\mathbf{R}_\mathbf{r}^{-1}\mathbf{s}_1}$$ equivalent to MMSE $a_1\mathbf{R}_\mathbf{r}^{-1}\mathbf{s}_1$

Adaptive realization (BLMS)

Blind!!

Inferior Performance

$$\varepsilon(n) = -\mathbf{c}^t(n-1)\mathbf{r}(n)$$

$$\mathbf{c}(n) = \mathbf{c}(n-1) + \mu\varepsilon(n)\left(\mathbf{r}(n) - \mathbf{s}_1^t\mathbf{r}(n)\mathbf{s}_1\right),\ \mathbf{c}(0) = \mathbf{s}_1$$

Linear constraint MMSE detectors

$$\hat{b}_1(n) = \operatorname{sgn}\left(\mathbf{c}^t \mathbf{r}(n)\right) = \operatorname{sgn}\left(\delta\, \mathbf{c}^t \mathbf{r}(n)\right), \;\; \delta > 0$$

Ambiguity in **c** for detection. Can be eliminated by imposing a constraint. We use the same constraint as in the BLMS

$$\mathbf{c}^t \mathbf{s}_1 = 1$$

Constraint MMSE problem

$$\min_{\mathbf{c}} E\left\{\left(v(n) - \mathbf{c}^t \mathbf{r}(n)\right)^2\right\}, \mathbf{c}^t \mathbf{s}_1 = 1$$

Adaptive realization. (We require $\mathbf{c}^t(n)\mathbf{s}_1 = 1$)

$$\varepsilon(n) = v(n) - \mathbf{c}^t(n-1)\mathbf{r}(n)$$

$$\mathbf{c}(n) = \mathbf{c}(n-1) + \mu\varepsilon(n)\left(\mathbf{r}(n) - \mathbf{s}_1^t\mathbf{r}(n)\mathbf{s}_1\right),\ \mathbf{c}(0) = \mathbf{s}_1$$

$$v(n) = \begin{cases} \alpha\, b_1(n) & \text{trained} \\ \alpha\,\mathrm{sgn}\left(\mathbf{c}^t(n-1)\mathbf{r}(n)\right) & \text{decision directed} \end{cases}$$

$\alpha = \begin{cases} 0 \\ 1 \\ a_1 \end{cases}$

α	Trained	Decision directed
0	Blind LMS (BLMS)	BLMS
1	Constraint LMS (CLMS)	DD-CLMS
a_1	CLMS with amplitude information (CLMS-AI)	DD-CLMS-AI

Optimum

Performance analysis

$$\hat{b}_1(n) = \operatorname{sgn}\left(\mathbf{c}^t(n-1)\mathbf{r}(n)\right)$$

$$\mathbf{c}^t(n-1)\mathbf{r}(n) = a_1 b_1(n) + \mathbf{c}^t(n-1)\tilde{\mathbf{r}}(n)$$

$$\tilde{\mathbf{r}}(n) = a_2\mathbf{s}_2 b_2(n) + \cdots + a_K\mathbf{s}_K b_K(n) + \sigma\,\mathbf{n}(n)$$

As our performance measure we propose

Excess ISIR

$$J(n) = a_1^{-2} E\left\{\left(\mathbf{c}^t(n-1)\tilde{\mathbf{r}}(n)\right)^2\right\} - a_1^{-2}\mathbf{c}_o^t\mathbf{R}_{\tilde{\mathbf{r}}}\mathbf{c}_o$$

Analysis of Trained Scheme

$$\varepsilon(n) = \alpha b_1(n) - \mathbf{c}^t(n-1)\mathbf{r}(n)$$

$$\mathbf{c}(n) = \mathbf{c}(n-1) + \mu\varepsilon(n)\left(\mathbf{r}(n) - \mathbf{s}_1^t\mathbf{r}(n)\mathbf{s}_1\right),\ \mathbf{c}(0) = \mathbf{s}_1$$

Theorem 1:

$$\overline{\mathbf{c}}(n) = \left(\mathbf{I} - \mu\left(\mathbf{I} - \mathbf{s}_1\mathbf{s}_1^t\right)\mathbf{R}_{\tilde{\mathbf{r}}}\right)\overline{\mathbf{c}}(n-1),\ \overline{\mathbf{c}}(0) = \mathbf{s}_1$$

$$J_{tr}(\infty) \approx a_1^{-2}\frac{\mu}{2}\left(\sum_{i=2}^{K} a_i^2 + (N-1)\sigma^2\right)\left(\sigma^2 + (\alpha - a_1)^2\right)$$

J_{tr} is minimum for $\alpha = a_1$, meaning that CLMS-AI is optimum.

Analysis of DD-CLMS-AI

$$\varepsilon(n) = a_1 \operatorname{sgn}\left(\mathbf{c}^t(n-1)\mathbf{r}(n)\right) - \mathbf{c}^t(n-1)\mathbf{r}(n)$$

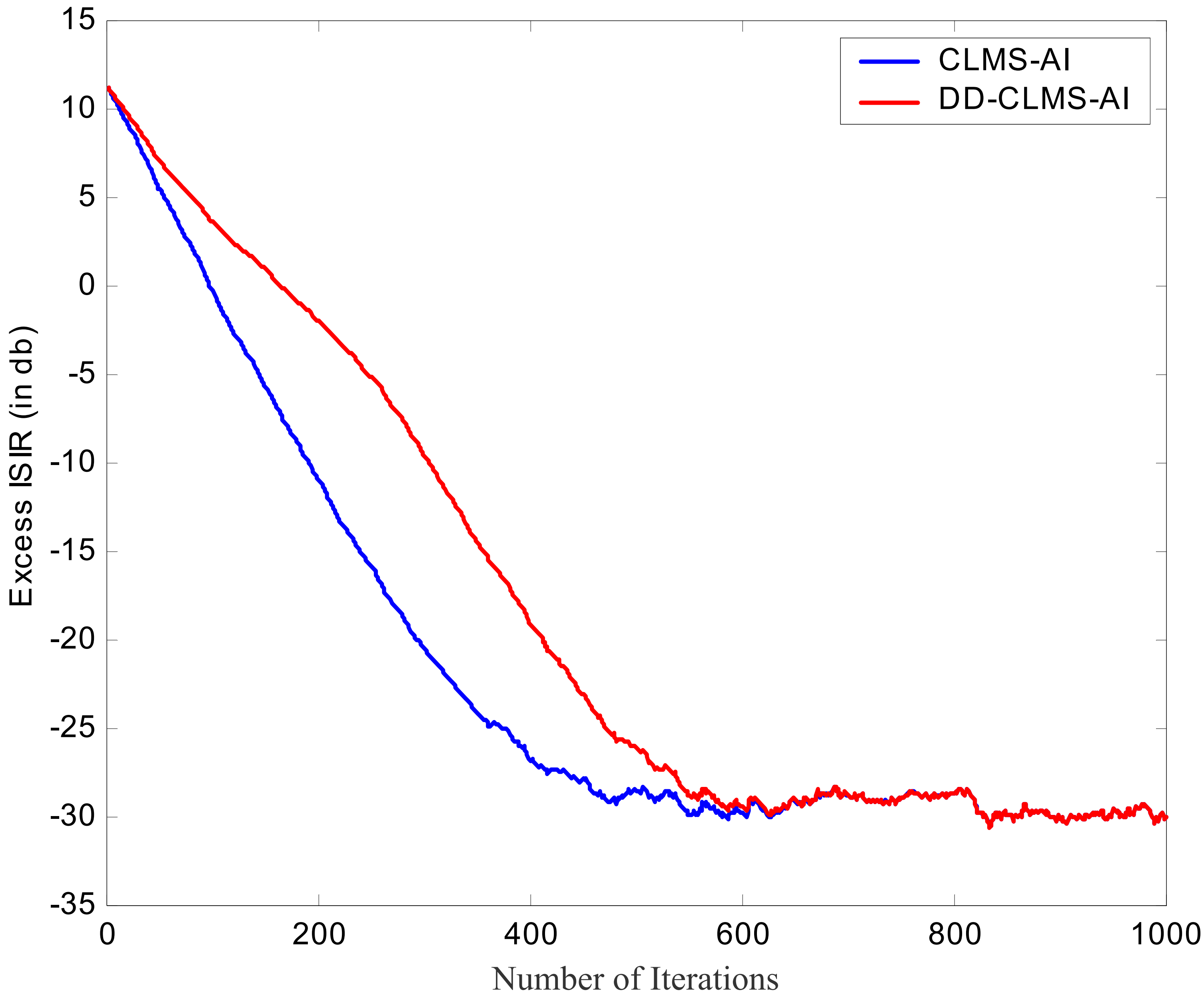
$$\mathbf{c}(n) = \mathbf{c}(n-1) + \mu\varepsilon(n)\left(\mathbf{r}(n) - \mathbf{s}_1^t\mathbf{r}(n)\mathbf{s}_1\right),\ \mathbf{c}(0) = \mathbf{s}_1$$

Theorem 2:

$$\overline{\mathbf{c}}(n) = \left(\mathbf{I} - \mu\left(\mathbf{I} - \mathbf{s}_1\mathbf{s}_1^t\right)\mathbf{R}_{\tilde{\mathbf{r}}}\,\rho\left(\frac{a_1}{\left\|\mathbf{R}_{\tilde{\mathbf{r}}}^{1/2}\overline{\mathbf{c}}(n-1)\right\|}\right)\right)\overline{\mathbf{c}}(n-1),\ \overline{\mathbf{c}}(0) = \mathbf{s}_1$$

$$\rho(x) = 1 - \sqrt{2/\pi}\, x e^{-x^2/2}, \quad 0.706 \le \rho(x) \le 1$$

$$J_{\mathrm{DD}}(\infty) \approx \frac{\mu}{2}\left(\sum_{i=2}^{K} a_i^2 + (N-1)\sigma^2\right)\left(\frac{\sigma^2}{a_1^2} + 4Q\left(\frac{a_1}{\sigma}\right) - \frac{4}{\sqrt{2\pi}}\frac{\sigma}{a_1}e^{-a_1^2/2\sigma^2}\right)\rho^{-1}\left(\frac{a_1}{\sigma}\right)$$



CLMS-AI
DD-CLMS-AI
Excess ISIR (in db)
Number of Iterations
15
10
5
0
-5
-10
-15
-20
-25
-30
-35
0
200
400
600
800
1000

Estimation of amplitude a_1

Let vector $\mathbf{c}$ be such that $\mathbf{c}^t\mathbf{s}_1=1$, then

$$E\left\{\mathbf{c}^t\mathbf{r}(n)b_1(n)\right\} = a_1 + E\left\{\sum_{i=2}^{K} a_i\left(\mathbf{c}^t\mathbf{s}_i\right)b_i(n)b_1(n) + \sigma\mathbf{c}^t\mathbf{n}(n)b_1(n)\right\}$$

$$= a_1$$

Trained

$$\hat{a}_1(n) = (1-\nu)\hat{a}_1(n-1) + \nu\mathbf{c}^t(n-1)\mathbf{r}(n)b_1(n),\ \hat{a}_1(0) = 0;$$

Decision Directed

$$\hat{a}_1(n) = (1-\nu)\hat{a}_1(n-1) + \nu\left|\mathbf{c}^t(n-1)\mathbf{r}(n)\right|,\ \hat{a}_1(0) = 0;$$

CLMS-AE (trained)

$$\hat{a}_1(n) = (1-\nu)\hat{a}_1(n-1) + \nu \mathbf{c}^t(n-1)\mathbf{r}(n) b_1(n),\ \hat{a}_1(0) = 0$$

$$\varepsilon(n) = \hat{a}_1(n) b_1(n) - \mathbf{c}^t(n-1)\mathbf{r}(n)$$

$$\mathbf{c}(n) = \mathbf{c}(n-1) + \mu\varepsilon(n)\left(\mathbf{r}(n) - \mathbf{s}_1^t\mathbf{r}(n)\mathbf{s}_1\right),\ \mathbf{c}(0) = \mathbf{s}_1$$

DD-CLMS-AE (blind)

$$\hat{a}_1(n) = (1-\nu)\hat{a}_1(n-1) + \nu\left|\mathbf{c}^t(n-1)\mathbf{r}(n)\right|,\ \hat{a}_1(0) = 0;$$

$$\varepsilon(n) = \hat{a}_1(n)\,\mathrm{sgn}\left(\mathbf{c}^t(n-1)\mathbf{r}(n)\right) - \mathbf{c}^t(n-1)\mathbf{r}(n)$$

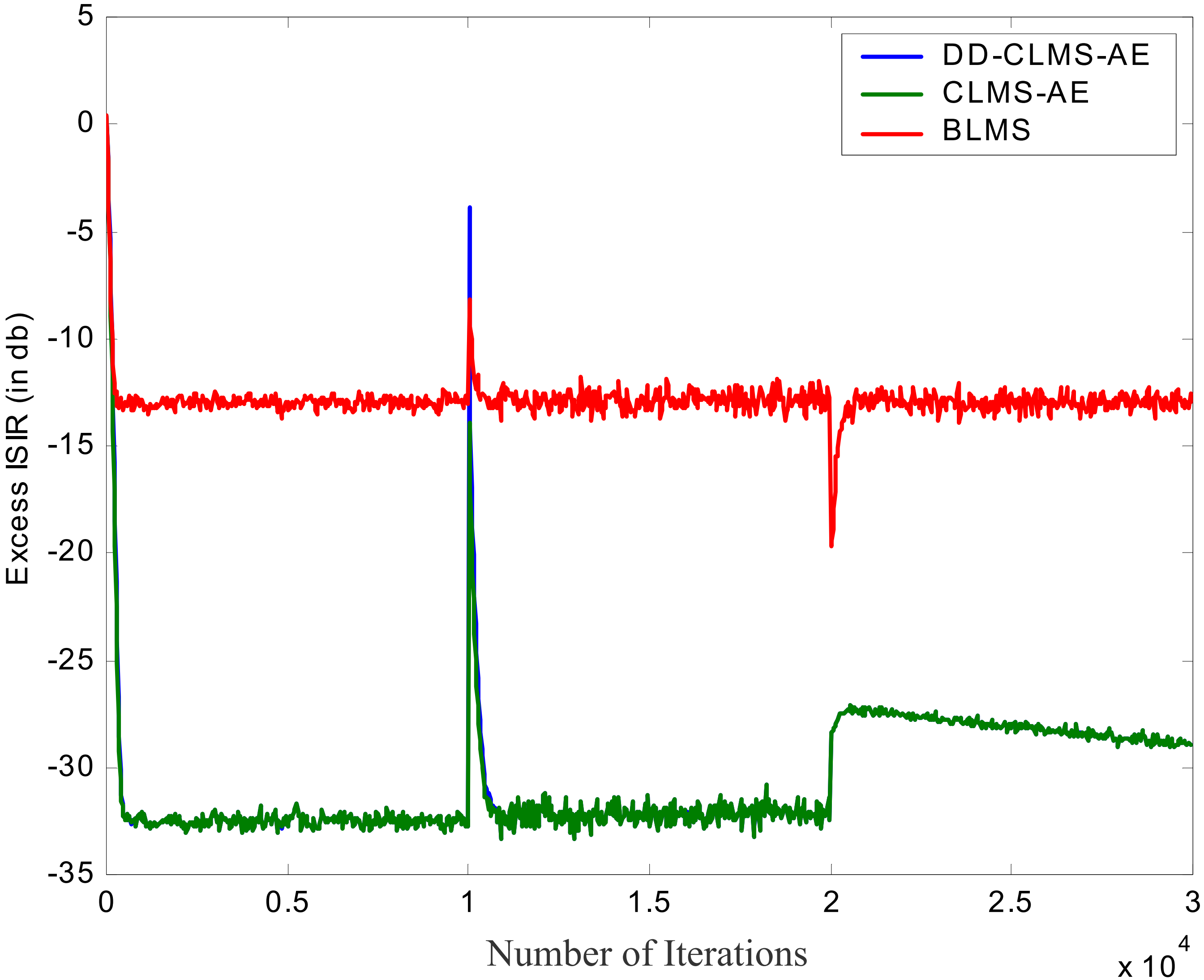
$$\mathbf{c}(n) = \mathbf{c}(n-1) + \mu\varepsilon(n)\left(\mathbf{r}(n) - \mathbf{s}_1^t\mathbf{r}(n)\mathbf{s}_1\right),\ \mathbf{c}(0) = \mathbf{s}_1$$

BLMS (blind)

$$\varepsilon(n) = -\mathbf{c}^t(n-1)\mathbf{r}(n)$$

$$\mathbf{c}(n) = \mathbf{c}(n-1) + \mu\varepsilon(n)\left(\mathbf{r}(n) - \mathbf{s}_1^t\mathbf{r}(n)\mathbf{s}_1\right),\ \mathbf{c}(0) = \mathbf{s}_1$$

Simulations

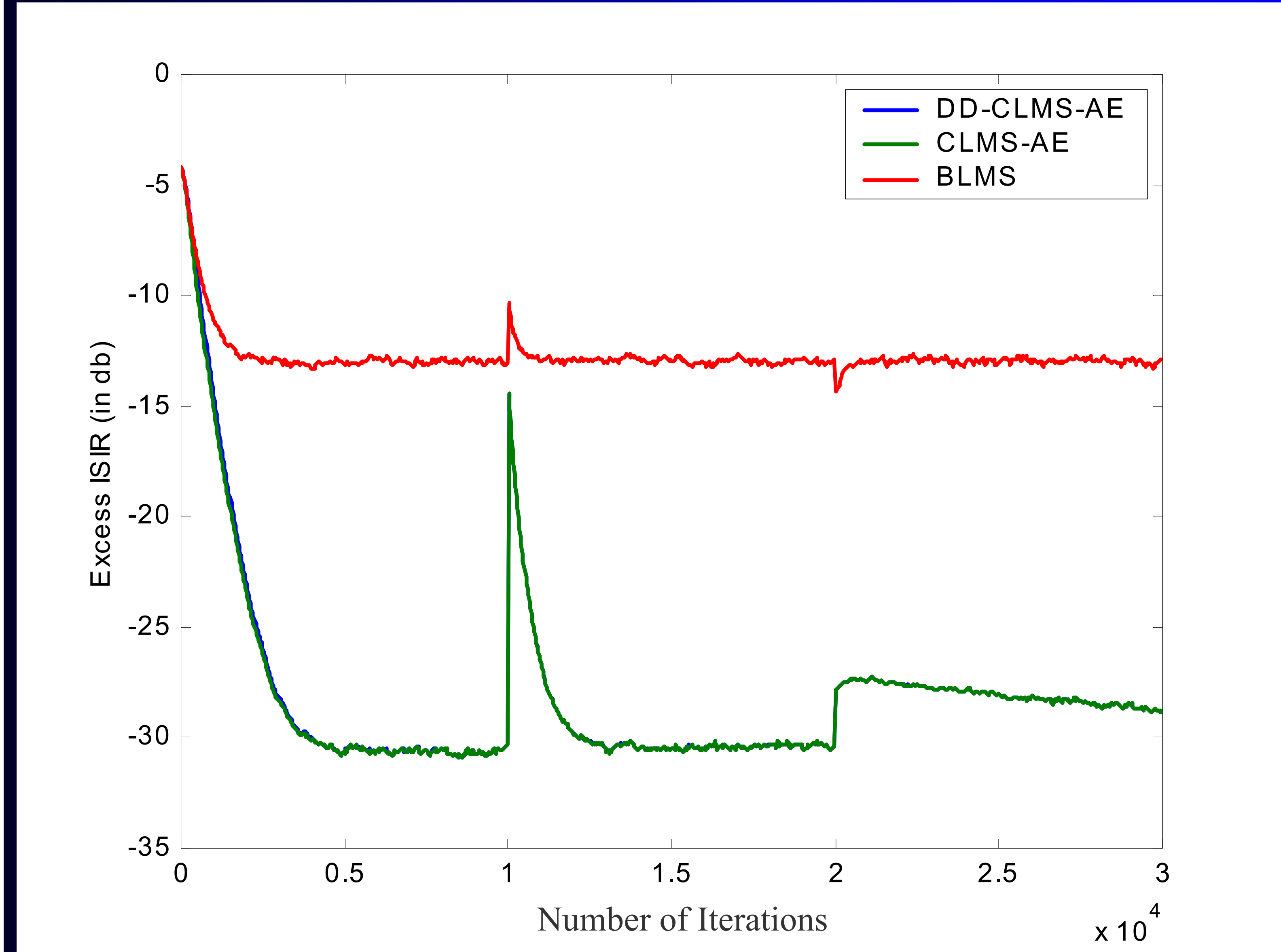


$N=128$

$K=7$
6, 10db

$K=8$
6, 10db
1, 20db

$K=4$
3, 10db

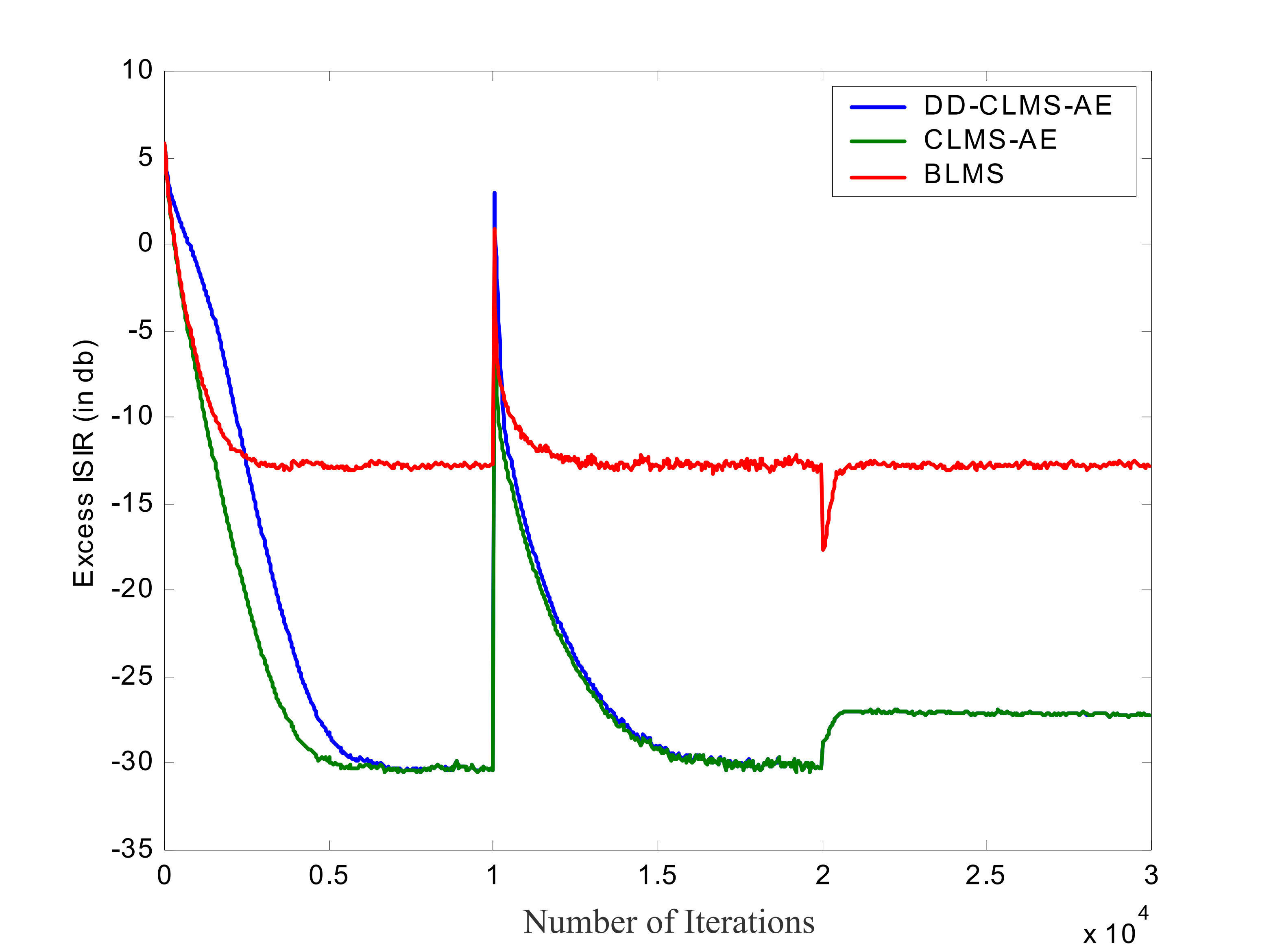


N=128

K=30
29, 0db

K=35
34, 0db

K=25
24, 0db



N=128

K=30
29, 10db

K=35
29, 10db
5, 20db

K=25
24, 10db

Conclusion

- We have presented a class of constraint adaptive algorithms suitable for the multiuser detection problem.
- We have analyzed the performance of the constraint algorithms and shown that the algorithm with amplitude information (CLMS-AI) is optimum while its decision directed version (DD-CLMS-AI) has slightly inferior performance.
- We proposed simple algorithms to estimate the amplitude of interest.
- Combinations of amplitude estimation algorithms with filter estimation algorithms were seen to be extremely efficient. Specifically the decision directed version (DD-CLMS-AE),which requires the same amount of a-priori information as the popular BLMS was seen to significantly outperform it.

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